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## DEBATE

### A new inequality index?

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#### Abstract

This article aims to motivate the debate on methods of measuring income inequality and poverty to compare regions in a country (or to compare selected countries). It is a criticism of an article published in volume 2 of the Brazilian Journal of Social and Labour Economics. The modification of the Gini index proposed by Esparza *et al.* (2020) involves arbitrary and unjustified definitions. There are already established methods, such as the generalized Lorenz curves, to compare the distribution of income in several regions without the analysis being limited to their inequality.

**Keywords:** Inequality; Income distribution; Poverty; Generalized Lorenz curve.

**JEL:** C10, D31, I32.

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Editor's note: The critical analysis published here is addressed to Esparza, Ortiz, Macías, & Maza's article (2020), "Bilateral Gini index: application for regional studies and international comparisons". RBEST offers these authors the right to reply.

## Um novo índice de desigualdade?

### Resumo

Este artigo tem como objetivo motivar o debate sobre métodos de mensuração da desigualdade de renda e da pobreza para a comparação de regiões de um país (ou para comparar países selecionados). Trata-se de uma crítica a um artigo publicado no volume 2 da Revista Brasileira de Economia Social e do Trabalho. A modificação do índice de Gini proposta por Esparza *et al.* (2020) envolve definições arbitrárias e sem justificativa. Já existem métodos consagrados para comparar a distribuição da renda em várias regiões, sem que a análise se limite à desigualdade, como, por exemplo, as curvas de Lorenz generalizadas.

**Palavras-chave:** Desigualdade; Distribuição de renda; Pobreza; Curva de Lorenz generalizada.

## ¿Un nuevo índice de desigualdad?

### Resumen

Este artículo tiene por objetivo motivar el debate sobre los métodos de medición de la desigualdad de ingresos y la pobreza para comparar regiones de un país (o comparar países seleccionados). Esta es una crítica a un artículo publicado en el volumen 2 de la Revista Brasileira de Economia Social e do Trabalho. La modificación del índice de Gini propuesta por Esparza *et al.* (2020) implica definiciones arbitrarias e injustificadas. Ya existen métodos establecidos para comparar la distribución de ingreso en diversas regiones sin que el análisis se limite a la desigualdad, como, por ejemplo, las curvas de Lorenz generalizadas.

**Palabras clave:** Desigualdad; Distribución de ingreso; Pobreza; Curva de Lorenz generalizada.

2

## Un nouvel indice des inégalités?

### Résumé

Cet article vise à motiver le débat sur les méthodes de mesure des inégalités de revenus et de la pauvreté pour comparer les régions d'un pays (ou pour comparer des pays sélectionnés). Il s'agit d'une critique d'un article publié dans le volume 2 de la Revista Brasileira de Economia Social e do Trabalho. La modification de l'indice de Gini proposée par Esparza *et al.* (2020) implique des définitions arbitraires et injustifiées. Il existe déjà des méthodes établies pour comparer la distribution des revenus dans différentes régions sans que l'analyse se limite aux inégalités, comme par exemple les courbes de Lorenz généralisées.

**Mots clés:** Inégalités; Répartition des revenus; Pauvreté; Courbe de Lorenz généralisée.

## Presentation and discussion of the proposal

In an article published in volume 2 of this Journal, Esparza *et al.* (2020) propose a new measure of inequality: a modification of the Gini index, which they call the “Bilateral Gini index”.

The incomes of a population of  $N$  people are indicated by  $x_1, x_2, \dots, x_N$ , with general average

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

Let us focus attention on a subset of this population, with  $n$  elements. If we have initially considered the population of a country, the subset may be, for example, the population of one of its regions. Let  $m$  indicate the number of values  $x_i \leq \mu$  and  $M$  represents the number of values  $x_i > \mu$  in this subset. Note that the subset values are compared with the overall population mean (not the subset mean). For every subset we have  $m + M = n$ . Let  $G$  indicate the Gini index of the subset.

According to the article, let us define  $p_M = M/n$  and the Bilateral Gini index is equal to

- (1)  $G_B = p_M G$  if  $0.5 < p_M < 1$ ,
- (2)  $G_B = p_M G - 1$  if  $p_M < 0.5$ ,
- (3)  $G_B = 0$  if  $p_M = 0.5$  and
- (4)  $G_B = 1 - G$  if  $p_M = 1$ .

Excluding the case of perfect equality ( $G = 0$ ), we have  $G_B > 0$  if  $p_M > 0.5$ , and obtain  $G_B < 0$  if  $p_M < 0.5$ .

If we are analysing the distribution of income in the regions of a country, the value of  $p_M$  in a region tends to be higher in relatively wealthy regions. It should be noted that

$$p_m = \frac{m}{n} = 1 - p_M$$

is the proportion of poor people in the region, if the general average income of the country is adopted as a poverty line. The “Bilateral Gini” mixes a measure of inequality with a measure of income level or poverty. If all incomes in a region are lower than the general average income of the population, then  $p_M = 0$  and  $G_B = -1$ , regardless of the value of  $G$ ; in this case the “Bilateral Gini” loses any relation to the inequality of income distribution in the region.

After that Esparza *et al.* (2020) define the Bidimensional Bilateral Gini index as the pair  $G$  e  $G_B$ . They state (p. 11), without any explanation, that this index obeys the Pigou-Dalton's condition. This condition establishes that the value of a measure of inequality should increase whenever a regressive transfer of income is made, that is, a transfer of income in which the recipient already has income equal to or greater than that of the donor. We know that the Gini index ( $G$ ) obeys the Pigou-Dalton's condition. It is necessary to define how the condition applies to a two-dimensional index: should the two components increase with the regressive transfer? But the "Bilateral Gini index" does not obey the Pigou-Dalton's condition, as shown in the following example.

Consider the subset of  $n = 4$  values 1, 3, 5, and 7 of a population with a general average  $\mu = 2$ . We can verify that  $p_M = 0.75$ ,  $G = 0.3125$  and  $G_B = 0.2344$ . By making two regressive transfers, the subset of 4 values becomes 1, 1, 1 and 13, with  $p_M = 0.25$ ,  $G = 0.5625$  and  $G_B = -0.8594$ .

Because the regressive transfers cause the value of  $p_M$  to fall from more than 0.5 to less than 0.5, the value of  $G_B$  changes from positive to negative, contrary to Pigou-Dalton's condition. In fact, the "Bilateral Gini" of Esparza *et al.* (2020) is not an appropriate measure of inequality and it is very inappropriate to denominate it using the name of the most usual of the inequality measures.

In the second paragraph of p. 21 of the article we can read that "the population within deciles IX and X concentrates 50% of the Mexican population, this helps us to visualize what the Bilateral Gini tries to contribute to the Gini Index: the decrease in the Gini value does not imply an improvement in the average income of the population." There is the usual confusion between "decile" and "tenth". The deciles are the nine values of the variable that divide the distribution into ten ordered tenths.<sup>1</sup> Anyway, it is absurd to say that two deciles or two tenths concentrate 50% of the population; two tenths obviously account for 20% of the population.

The rest of the sentence shows that the authors consider the use of the "Bilateral Gini" relevant because it shows that a decrease in the value of the Gini index of a population does not imply that the average income has increased. But who is the researcher who, when analysing Gini indices, stated that there is such a negative statistical association between variations in inequality and in average income?

<sup>1</sup> The word "percentile" was first defined by Galton (1885). A rigorous definition of percentiles and deciles can be found in Davidson and MacKinnon (2004, p. 180).

## Critical analysis of the proposal

Sorry for the tautology, but an index of inequality should measure inequality. It is not reasonable to pretend to create an index of “inequality” that captures all the characteristics of a distribution or, worse, all the socioeconomic characteristics of a region. The distinction between central tendency, dispersion and inequality of a distribution is well established in statistics and it is generally advisable to measure these characteristics separately. If the researcher wishes to compare the distribution of income in regions of a country taking into account both the average level of income and the inequality of the income distribution, he can, for example, analyse the values of the Gini index and the average income in each region. He could include poverty measures for various poverty lines. This would be much simpler and clearer than the use of the “Bilateral Gini”.

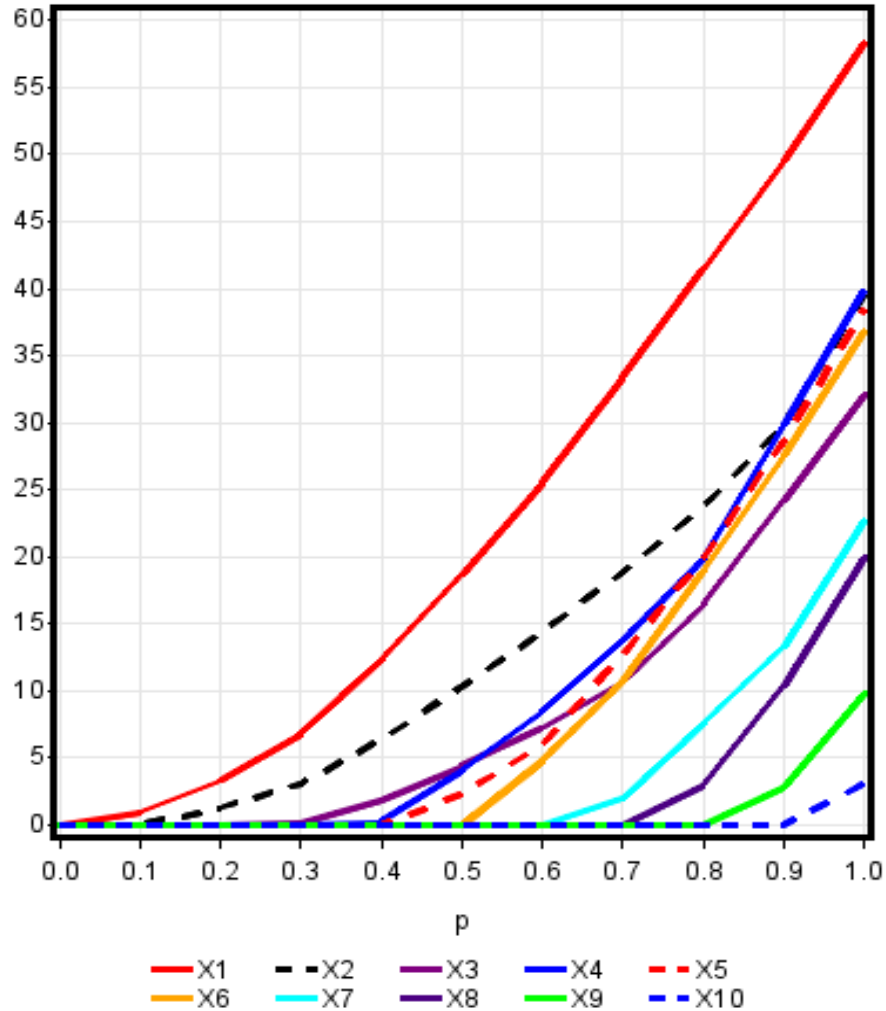
A more sophisticated way of comparing various distributions considering both the means and the inequalities is to compare the generalized Lorenz curves of these distributions (see Hoffmann, Botassio, & Jesus, 2019, pp. 280-286). At the beginning of p. 13 of the article, Esparza *et al.* (2020) present artificial data for ten sets of 10 values of a variable. Let us indicate these sets by  $X1, \dots, X10$ . These sets present increasing values of the Gini index, with  $G = 0,2403$  for  $X1$  and  $G = 0,9000$  for  $X10$ . Poverty tends to grow, which is manifested in the growing trend in the values of  $p_M$ . Using these data, we constructed the generalized Lorenz curves for the 10 sets, presented in Figure 1.

Note that the  $X1$  distribution dominates in second order all nine others. The  $X1$  distribution has the highest average income (58.3) and the lowest Gini index. The fact that the  $X1$  distribution dominates in second order the  $X2$  distribution, for example, means that the total well-being associated with  $X1$  is greater than that associated with  $X2$  for all increasing and Schur-concave social welfare functions of income; as the two distributions have the same number of elements (10), it also means that  $X1$  can be obtained from  $X2$  by making increases in incomes and/or progressive transfers. The fact that the generalized Lorenz curve for  $X1$  is always above the curve for  $X2$  ensures that the Foster, Greer, and Thorbecke's poverty measure with  $\alpha = 1$  (which is equal to the  $HI$  product between the proportion of poors and the income gap ratio)<sup>2</sup> in  $X1$  is always lower than in  $X2$ , whatever the poverty line adopted.

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<sup>2</sup> The  $HI$  product of the head count ratio  $H$  times the income insufficiency ratio  $I$  is a measure of poverty that captures both the extent and the intensity of poverty. For a didactic presentation of the subject, see chapter 11 of Hoffmann, Botassio, and Jesus (2019). Foster, Greer, and Thorbecke (2010) made a wide revision of the literature on poverty measures since the time of their original 1984 article, highlighting the importance of Sen's previous article (1976a).

Figure 1. Generalized Lorenz curves for the 10 subsets  
(table data in p. 13 of the article by Esparza *et al.*, 2020)



6

Note the crossing between the  $X3$  and  $X6$  curves. There is no second-order dominance between these two distributions. The mean is higher in  $X6$  (36.8) than in  $X3$  (32.1), but the Gini index is higher for  $X6$  ( $G = 0.5636$ ) than for  $X3$  ( $G = 0.4938$ ).

If the objective is to classify several regions considering both the inequality and the central tendency of income distributions, another alternative is to use the welfare measure  $W_h = \mu_h(1 - G_h)$ , where  $\mu_h$  and  $G_h$  are, respectively, the average income and the Gini index of the  $h$ -th region. Sen (1973/1997) and Sen (1976b) are fundamental references in the broad literature on the subject.

In Table 1 of the article, Esparza *et al.* (2020) list some measures of inequality and indicate their properties. The first two in the table are the variance and the standard deviation. Strictly speaking, the variance and the standard deviation are measures of dispersion, not of

inequality. The coefficient of variation is a measure of inequality and its formula makes it clear that it is a measure of *relative* dispersion. If a person considers that the standard deviation of income is a measure and inequality, he should agree with the statement that the monetary changes that occurred in Brazil, with the monetary unit becoming 1,000 times greater, led, at the time, the inequality of income distribution in the country to be reduced to one thousandth of its former value.

It is stated, in the table, that the variance of logarithms obeys the condition of Pigou-Dalton. This is not true (see Hoffmann, Botassio, and Jesus, 2019, pp. 174-177).

Conclusion: inappropriate innovations and errors make it advisable to forget this article by Esparza *et al.* (2020).

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