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DOSSIER: INEQUALITY

Bilateral Gini index:
Application for regional studies and international comparisons

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Abstract

In this paper, we have considered a new contextualization of the Gini index, giving a particular interpretation of inequality. The Gini index is the most widely used measure of inequality in the world, however, it does not meet some desirable properties of an inequality indicator. Even so, as it is a measure adopted by most countries through the years, makes it a valuable statistical input, which requires adjustments that provide information, making the study of inequality more robust by adding different indicators that can account for their economic, political, and social environment. This article provides a variation of the Gini index with the purpose of compare and classify different territories (the States of Mexico, as well as selected countries) with similar Gini index. The classification is carried out into groups (called turbines) with either positive equality, negative equality, positive inequality or negative inequality. The main contribution of this paper lies on distinguish territories with similar Gini index but different average income, from what can be inferred the conditions of each one and how privileges and facilities are distributed into them.

Keywords: Income inequality; Gini index; Regional inequality; México.
JEL: C10, D31, E25, O15, P52.

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Índice de Gini bilateral:
Aplicação para estudos regionais e comparações internacionais

Resumo

Neste artigo, apresentamos uma abordagem alternativa do índice de Gini, oferecendo uma interpretação particular da desigualdade. O índice de Gini é a medida de desigualdade mais usada no mundo, no entanto, não possui algumas propriedades desejáveis de um indicador de desigualdade. Embora seja uma valiosa ferramenta estatística, requer ajustes para fornecer informações e tornar o estudo da desigualdade mais robusto ao agregar diversos indicadores que possam retratar aspectos econômicos, políticos e sociais. Este artigo apresenta uma variação do índice de Gini com o objetivo de comparar e classificar diferentes territórios (os estados do México, bem como países selecionados) com índice de Gini semelhante. A classificação é realizada por meio de grupos ("turbinas") com igualdade positiva, igualdade negativa, desigualdade positiva ou desigualdade negativa. A principal contribuição do artigo reside em distinguir territórios com índice de Gini semelhante, mas renda média diferente, a partir do que se pode inferir as condições de cada um e a distribuição de privilégios e facilidades entre eles.

Palavras-chave: Desigualdade de renda; Índice de Gini; Desigualdade regional; México.

Índice de Gini Bilateral:
Aplicación para estudios regionales y comparaciones internacionales

Resumen

En este trabajo hemos considerado una nueva contextualización del índice de Gini, dando una interpretación particular de la desigualdad. El índice de Gini es la medida de desigualdad más utilizada en el mundo; sin embargo, no cumple con algunas propiedades deseables de un indicador de desigualdad. Aun así, al ser una medida adoptada por la mayoría de los países a lo largo de los años, la convierte en un insumo estadístico valioso, que requiere de ajustes que brinden más información, fortaleciendo el estudio de la desigualdad al agregar diferentes indicadores que puedan dar cuenta de sus características económicas, políticas y el entorno social. Este artículo presenta una variación del índice de Gini con el propósito de comparar y clasificar diferentes territorios (los Estados de México, así como países seleccionados) con índice de Gini similar. La clasificación se lleva a cabo en grupos (llamados turbinas) con igualdad positiva, igualdad negativa, desigualdad positiva o desigualdad negativa. El principal aporte de este trabajo radica en distinguir territorios con índice de Gini similar pero diferente ingreso promedio, de lo que se puede inferir las condiciones de cada uno y cómo se distribuyen en ellos privilegios y facilidades.

Palabras clave: Desigualdad del ingreso; Índice de Gini; Desigualdad regional; México.

Indice de Gini bilatéral:
Utilisation pour les études régionales et les comparaisons internationales

Résumé

Dans cet article, nous présentons une approche alternative à l'indice de Gini, offrant une interprétation particulière de l'inégalité. L'indice de Gini est la mesure de l'inégalité la plus largement utilisée dans le monde, mais il n'a pas certaines propriétés souhaitables d'un indicateur d'inégalité. Bien qu'il s'agisse d'un outil statistique précieux, il nécessite des ajustements pour fournir des informations et rendre l'étude des inégalités plus robuste en agrégeant divers indicateurs susceptibles de présenter des aspects économiques, politiques et sociaux. Cet article présente une variation de l'indice de Gini afin de comparer et de classer différents territoires (les États du Mexique, ainsi que des pays sélectionnés) avec un indice de Gini similaire La classification est effectuée au moyen de groupes (appelée turbines) avec égalité positive, égalité négative, inégalité positive ou inégalité négative La contribution principale de l'article consiste à distinguer les territoires avec un indice de Gini similaire, mais des revenus moyens différents, à partir desquels on peut déduire les conditions de chacun et la répartition des privilèges et des facilités entre eux.

Mots clés: L'inégalité des revenus; Indice de Gini; Inégalité régionale; Mexique.

Introduction

The Gini coefficient was proposed by Corrado Gini in 1914, since then it is considered the most popular measure of inequality used worldwide. It is important to note that this is not a measure of wealth, poverty, or general conditions of an economy, but rather a concentration of a variable.

Gini based part of his work on the theory of the "average man" (Quételet, 1848/ 2018), which postulates an association between biological and social normality, with the frequency of appearance of certain population features (Gini, 1914). Although it is not the subject for this article, it is of the utmost importance to put the Gini indicator in context because of the ideological content associated with a political and historical moment, in which many contributions to science had the explicit purpose of justifying the interests of a political agenda, specifically, in this case, the one imposed by Benito Mussolini (Favero, 2004).

Therefore, the formulation of the Gini coefficient is based on an inequality respect to an average taking into account Lorenz's contribution in inequality subject; Lorenz (1905) justified the importance of having a measure of income inequality, regardless of what "inequity in the distribution of wealth" means, such as the measure gives a concentration variable with values over a range from 0 (perfect equity) to 1, indicating the inequality respect to an average (equidistribution line). This average value is not available and cannot be deducted, therefore a same Gini value can be produced from different average incomes, population size, and even with relevant differences in the income distribution itself (Ackermann & Cortés, 1979; Selita & Kovas, 2019).

It is important to note that the extreme values taken by the Gini coefficient, in the case of income, are hypothetical, i.e., no capitalist economy will obtain a Gini coefficient close enough to 0 or 1 (Anand & Segal, 2008; Milanović, 2011), since the capitalist production system feeds on itself (Piketty, 2015a), so it will always be unequal, in addition, there is no "ideal value" of the coefficient. Therefore, there can be two economies with the same value of the Gini coefficient and still have diametrically different living conditions, or a constant Gini value over time for the same economy, in which the average income has increased/decrease through the years.

The main advantage of the Gini coefficient is that it is a measure of the inequality that it is obtained through an analysis of reasons. This index does not consider the size of the economy, the way it is measured, or if it is a rich or poor country on average.

Due to this, the Gini coefficient of a large and economically diverse country, generally results in a much higher coefficient than that of each of its regions has individually.

Also, it should be considered that the comparison of the distribution income between countries can be difficult because the benefit systems may differ. In fact, the measure will give different results when applied to individuals instead of houses.

As for all statistics, there will be systematic and random errors in the databases used for calculating the Gini coefficient. In addition, countries may collect data differently, which makes a difficult statistical comparison between countries, even though, there are organizations collecting comparable data. Economies with similar Gini coefficients can have very different income distributions, this is because the Lorenz curves can have different forms and the same Gini coefficient.

Therefore, this paper proposes an extension of the Gini index within a particular context, which in turn considers the Gini coefficient itself but also makes a very important distinction between rich and poor economies.

In Mexico, some researches highlight a problem in data collection and under-registration of data, which has important consequences in the calculation of inequality. Esquivel Hernández (2020) and Cortés and Vargas (2017) make adjustments to the measurement of the index of Gini by estimating the under-declarations of the income of the upper strata, which causes value to increase consistently.

The paper is organized as follows: in Section 1 we present some background of the Gini index and other measures of inequality. The proposed methodology of analyzing the Gini index is presented in Section 2, including a simulation database example. In Section 3, using real databases, we obtain the Bilateral Gini index. Final remarks are presented in Section 4.

1. Background

Frequently, Gini values are attempted to be associated with income distribution among classes, by grouping into income ranges, however, the Gini coefficient is not a measure in which this information can be inferred, since the division of deciles may not corresponds to the average income or to homogeneous conditions of classes in order to make them comparable. There are also studies linking high coefficient values to political instability in different countries, in which there is also no conclusive data (Tao, Wu, & Li, 2017), the World Bank generally considers a Gini value of 0.4 as alarming, value that in 2016, for example, would classify countries such as the United States (0.415), Turkey (0.419), or El Salvador (0.40) in almost the same category of inequality (World Bank, 2016).

Returning to the ideology in which the indicator was designed, it can be deduced that a desired value of the Gini should be close to “0” under the assumption that the income should be distributed equally among individuals, postulated close to the socialist ideology propagated by fascism, so it is paradoxical the adjustment of a "socialist suit" to a "capitalist body" in which wealth depends on the engrossment of the means of production and where inequality is found, since its genesis due to the principle of scarcity (Piketty, 2015b).

In addition, the idea of the average man was designed for an "ideal average value", therefore, in the case of the distribution of wealth, implies that there is a social agreement on what a sufficient income means, consequently, we can assume that this value should be a decent, sufficient average income that guarantees well-being; conditions that low-income economies would not meet because there could be equality in poverty too.

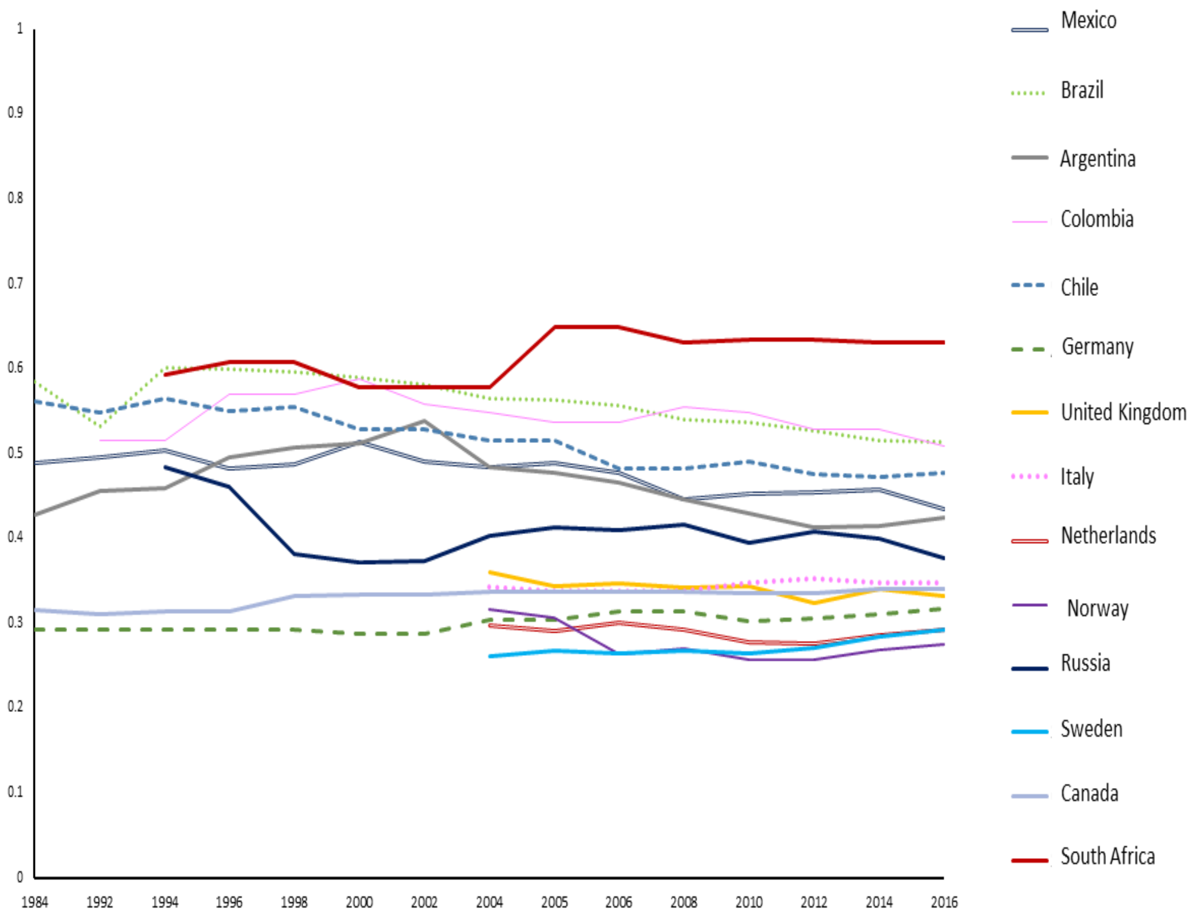
This represents a consistent difficulty in the comparability between Gini values worldwide. In order to counteract this problem, different methodologies have been made, being the Milanovic's proposal (Milanović, 2017) on the Purchasing Power Parity (PPP) of the income makes a comparison possible, however, it is affected by the problems related to different techniques of obtaining databases, the statistical resources of each country, even the different definitions of what income is. Complications that, although less visible, should also be considered even for comparisons within regions of the same country.

Furthermore, we have to consider the stability of the values of Gini over time, due to the range of the values that the coefficient have historically taken, as Milanović (2017) pointed out: income inequality has prevailed over the last two centuries. For example, Figure 1 shows the Gini coefficient of different countries from 1984 to 2016, in which, Gini's values are around 0.2 to 0.7, however values of each country show practically horizontal lines in this period, with the exception of Russia and South Africa that had significant decrease/increase respectively in the Gini value of approximately 0.1.

On the contrary, less unequal countries, such Sweden or Norway, do not show significant variations in 15 years, even if we think about the migratory movements they have had and from which it would be expected that an inequality measure would be able to capture, this may suggest that it is necessary to define the implication of the micro variations in the value of the Gini coefficient in real inequality, as well as the relationship that may exist between inequality and demographic characteristics such as population size.

From Figure 1, it is possible to ask whether the guidance of governments and therefore their public policies have been effective in reducing inequalities, because from the information generated by the Gini coefficient, in 30 years in countries with the most diverse forms of organization; open, closed, populist, neoliberal, right, center or left economies; crossed by wars, crises, expansions and economic contractions are not able to consistently reduce inequality.

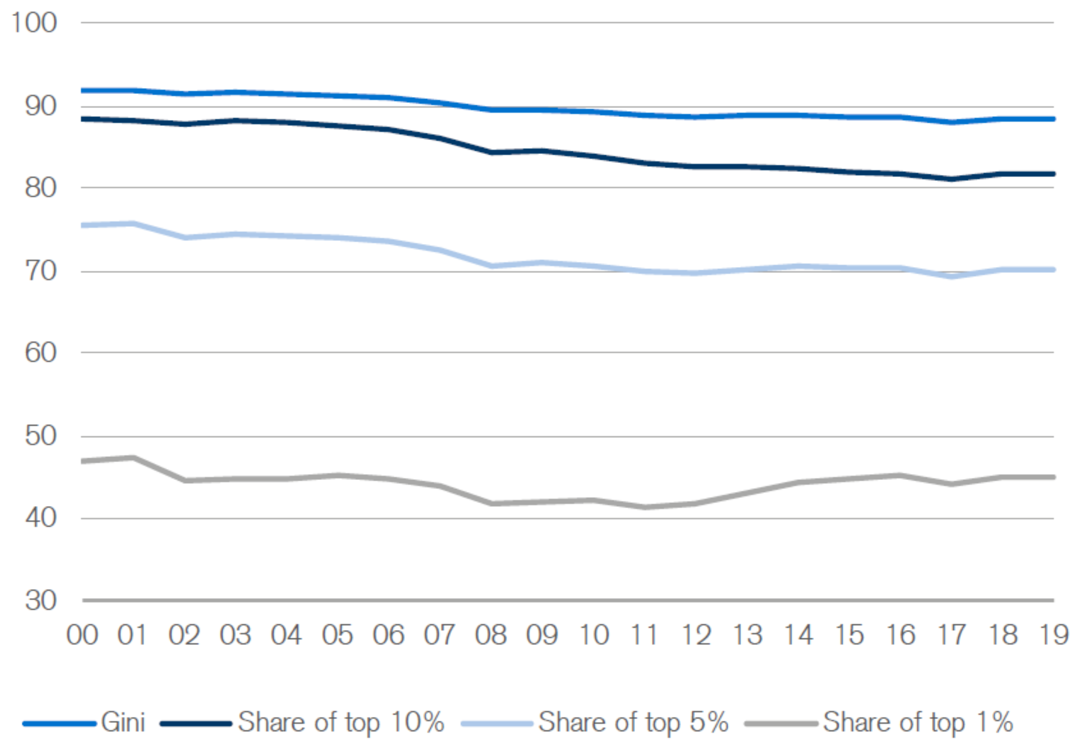
Figure 1. Gini Coefficient values for different countries, 1984-2016.



Source: World Bank Data (Escobedo, 2020).

Thus Gini's values of any country have not significant changes over time, only slight variations, therefore inequality is systemic, and perhaps needs modifications in the parameters that limit the maximum and minimum values only to delimit that there are less voracious forms of capitalism, in the sense that may be better income redistribution in some countries, associated, for example, with quality in the State services via taxes; however, data indicate that in all cases, the profit rates of the upper strata (which can match the holders of capital) are equivalent, even in the least unequal countries (Credit Suisse Research Institute, 2019). That is, capitalism is voracious in all economies. The only thing that changes is the distribution in the middle and lowers layers of the population. According to the Global Wealth Report (2019), the value of the Gini coefficient of the top 5% of the richer population worldwide remain stable in the past 10 years with slight variations passing from 0.706 in 2008 to 0.702 in 2019 (see Figure 2).

Figure 2. Share of the top 10%, 5% and 1% of the world population in wealth distribution 2000-2019.



Source: Global Wealth Report 2019 (Credit Suisse Research Institute, 2019).

1.1. Inequality measures

In the literature there are many inequality measures. According to Cowell (2011) inequality measures must have three fundamental properties (Atuesta, Mancero, & Tromben, 2018):

1. Principle of Transfers (Pigou-Dalton): Transfers of individuals at the top of the distribution to individuals in the lower part of the income distribution reduce the measure of inequality. This property means that an inequality index must assign different weights to each income based on where they are in the income distribution.

2. Additive decomposition: The measure of inequality can be decomposed by population subgroups. It is particularly desirable an additive separability, i.e., the value of the index for the whole population can be obtained as the sum of the intra-group and inter-group inequalities of the subgroups.

3. Scale independence: If the analyzed variable is multiplied by the same scalar for all individuals in the population, the degree of inequality does not vary.

In Table 1 we present some of inequality measures and their features in order to compare with the Gini index.

Table 1. Inequality measures comparison.

Measure	Description	Advantages	Disadvantages
Variance	It measures how far the observations are with respect to the average.	Simplicity in its calculation. Satisfies the condition of Pigou-Dalton.	Does not satisfy the property of independence of the average, which is not desirable as an indicator of inequality.
Standard deviation	It is defined as the square root of the variance. Measures the average distance of the distribution mean, but in the same units in which the average is calculated.	Simplicity in its calculation.	Does not satisfy the scale invariants (their results change along with the average value of the variable), this could cause that a distribution has a smaller variance than another, despite presenting a greater relative variation, if the average income of the first distribution is less than that of the second.
Coefficient of variation	It refers to the relationship between the size of the mean and the variability of the variable.	It is independent of the average income level. It is sensitive to any income transfer in the distribution.	The sensitivity to transfers does not depend on the value of the income.
Logarithmic variance and standard deviation of logarithms	It places greater importance to income transfers generated at the bottom of the distribution.	Satisfies the property of Pigou-Dalton. Satisfies all desirable properties including transfer sensitivity property.	Inconsistent with the ownership of transfers in the upper part of the distribution, which causes that the transfers from the very rich to the less rich an increment of the concentration instead of reducing it.
Lorenz Curve	Illustrates the inequality in the distribution. Represents the cumulative percentage of the income received by a certain group of the population which are ordered in an ascending way according to the amount of their income.	Intuitive.	For its construction only takes the percentages of the population and income, which isolates the effect of total income and, therefore, it only reflects the structure and form of the distribution.
Gini coefficient	Measures the relative inequality of a population.	It is the most used index to measure inequality. Satisfies the Pigou-Dalton property: any transfer from one individual to another with a lower level is reflected in a drop in the indicator. Satisfies the properties of invariance to scale, invariance to replicas and symmetry.	It does not satisfy the transfer sensitivity property. Does not satisfy the property of decomposition.

Schutz Index (Robin Hood Index)	Measures the maximum vertical distance between the Lorenz curve and the equidistribution line. It can be interpreted as the proportion of the income that has to be transferred to people whose income is less than the average so that their income is equal to the average and it reaches the perfect equality.	It is also based on Lorenz's work. Simplicity on its calculation.	Does not satisfy the Pigou-Dalton principle.
Theil Index	It measures inequality based on the concept of entropy, derived from the information theory which describes how much randomness there is in an event; the degree of entropy of an event is a decreasing function of its probability of occurrence.	It satisfies all desirable properties for an inequality indicator, in addition to satisfying the additive decomposition property.	
Atkinson Index	It is a parameter that measures the inequality of the income distribution in a society emphasizing the subgroups that compose it.	Easy to interpret. Satisfies the property of Pigou-Dalton and the property of sensitivity to transfers.	

Source: Own elaboration based on Medina (2001).

In Table 2, we summarize some inequality measures along with some properties.

Table 2. Properties of inequality measures.

Inequality indicator	Transfer principle	Additive decomposition	Independence of scale, income, and population size
Variance	Strong	Yes	No, it increases with income value
Coefficient of variation	Weak	Yes	Yes
Gini	Weak	No*	Yes
Atkinson	Weak	Yes	Yes
Dalton	Weak	Yes	No
Theil	Strong	Yes	Yes

Source: Medina (2001).
*It is only satisfied if the ordering of the income groups matches that obtained from the total income.

The Gini coefficient does not have the decomposition property, so, from a social and economic point of view, it is a weak measure to infer the way income is distributed according to social class, gender or any variable that may contextualize the indicator. Despite the existence of indicators that offer this property, Gini coefficient has an advantage that no other measurement can offer: the availability and the possibility of using it cross-sectionally.

Since most economies calculate and track this indicator over time, in this article we will consider this measure of inequality in order to propose a new technique to interpret and make comparisons of Gini indices of different places. In particular, we want to add a "label" to the Gini inequality index. This label allows us to discriminate either rich or poor economies: if two countries have the same Gini inequality index, one would be better than the other if the percentage of poor people is lower.

2. Methodology

Let consider a finite set of agents $N := \{1,2,\cdots,n\}$ - the population - with income[1] given by the vector $\boldsymbol{x} := (x_1, x_2, \dots, x_n)$. The population mean is given by $\bar{x} = \sum_{i\in N} \frac{x_i}{n}$. Now, for each subset of the population $S \subset N$ with cardinality n_s, let define the following:

$$m(s) := \{i \in S | x_i \le \bar{x}\}, \quad \text{and} \quad M(s) := \{i \in S | x_i > \bar{x}\}$$

with cardinalities m_s and M_s, respectively. Note that $m_s + M_s = n_s$.

Thus, the proportion of the subset data less and more than the population mean are given by (respectively):

$$pm := \frac{m_s}{n_s}, \quad \text{and} \quad pM := \frac{M_s}{n_s}$$

Given this notation we have the following definition.

Definition 1. Let $S \subset N$ be any subset of the population such that $\boldsymbol{y} = (y_1, y_2, \dots, y_{n_s})$ is the income vector with elements in S. Being $G(\boldsymbol{y})$ the Gini index of this subset data, the Bilateral Gini ($BG(\boldsymbol{y})$) index is defined as follows:

1. If $M_s > m_s$ and $M_s \neq n_s$:

$$BG(\boldsymbol{y}) := pM \cdot G(\boldsymbol{y}).$$

2. If $M_s < m_s$:

$$BG(\boldsymbol{y}) := pM \cdot G(\boldsymbol{y}) - 1.$$

3. If $M_s = m_s$:

$$BG(\boldsymbol{y}) := 0.$$

In this case, $pm = pM = 0.5$.

[1] We use income as a variable that contextualizes in a general way and as an explanatory example of the Bilateral Gini methodology, however, we can introduce other variables, e.g., those needed in the quality of life index, those included in indicators of well-being, in order to strengthen the results, and thus have an indicator more grounded on the systemic differences of the territories and the values of the Gini coefficient.

4. If $M_s = n_s$:

$$BG\ (\boldsymbol{y}) := 1 - G(\boldsymbol{y}).$$

In this case, $pM = 1$.

In a compact form, can be written as:

$$BG(\boldsymbol{y}) = pM \cdot G(\boldsymbol{y})1_{\{M_s > m_s\}} + (pM \cdot G(\boldsymbol{y}) - 1)1_{\{M_s < m_s\}} + \left(1 - G(\boldsymbol{y})\right)1_{\{M_s = n_s\}}$$

where $1_{\{\cdot\}}$ is the indicator function.

Once defining the Bilateral Gini, we can define the following bidimensional index.

Definition 2. The Bidimensional Bilateral Gini index (B) is defined as follows:

$$B(\boldsymbol{y}) = (G(\boldsymbol{y}), BG(\boldsymbol{y})) \text{ where } \boldsymbol{y} = (y_1, y_2, \dots, y_{n_s}).$$

Note that this index satisfies the property of scale independence, i.e., $B(k\boldsymbol{y}) = B(\boldsymbol{y})$ for any constant k. Moreover, also satisfies the Pigou-Dalton property and symmetry.

Based on Tao, Wu, and Li (2017), let us assume an alarming level of the Gini of 0.5. We define the "Gini's windmill" such that having the following four turbines:

1. Positive Equality (PE): *Gini< 0.5, BG ≥ 0*.
2. Negative Equality (NE): *Gini < 0.5, BG < 0*.
3. Positive Inequality (PI): *Gini ≥ 0.5, BG ≥ 0*.
4. Negative Inequality (NI): *Gini ≥ 0.5, BG < 0*.

In the positively equality turbine, economies with Gini values less than 0.5 are considered, although, as previously explained, a value of 0.4 is also alarming in terms of inequality, however, this value of 0.5 serves to illustrate the example. In this turbine, it is also considered that the proportion of the subpopulation with higher incomes to the average income of the population is greater.

In this turbine, it would be expected to find stable economies, with low unemployment rates and high wages, with efficient social security systems and tax collection, making that the proportion of the population that have incomes below the average, has balance in terms of access to efficient public services. Nordic economies would be the perfect candidates for this quadrant, however, given the Gini parameter of 0.5, we could find some European and Asian economies as well.

Second turbine shows the negatively equality quadrant in which we have low Gini values and the majority of the population would have an income below the average (equality in poverty (Cortés & Rubalcava, 1991)), this phenomenon occurs especially in territories where economic conditions are bad for all the population (which makes them have a "low" Gini) but on the other hand, the income of the majority does not give to adjust to the average income.

This quadrant reveals an economic reality that would be hidden otherwise, that is, although it has an adequate level in the inequality index, it is not necessarily tied to a good standard of living, in this classification (given the Gini value less than 0.5), we would expect to find the majority of economies, however, if we took a Gini value less than 0.4, most economies would move to the quadrant of negatively inequality, that is, the bulk of the population would be expected to live in situations of inequality and poverty. In this turbine, we expect to find the so-called emerging economies of Asia and Latin America.

So as to the third turbine, it shows positive inequality which mean high level of inequality where the proportion of the subpopulation with incomes above the average income is greater, due to what was previously explained about the way the capitalist system is configured, this quadrant is not possible, given that there is no economy capable of producing more rich people than poor people. However, in the configuration of this paper some territories can be in this turbine, if their Gini index is more than 0.5 and its proportions of rich people are higher than poor people.

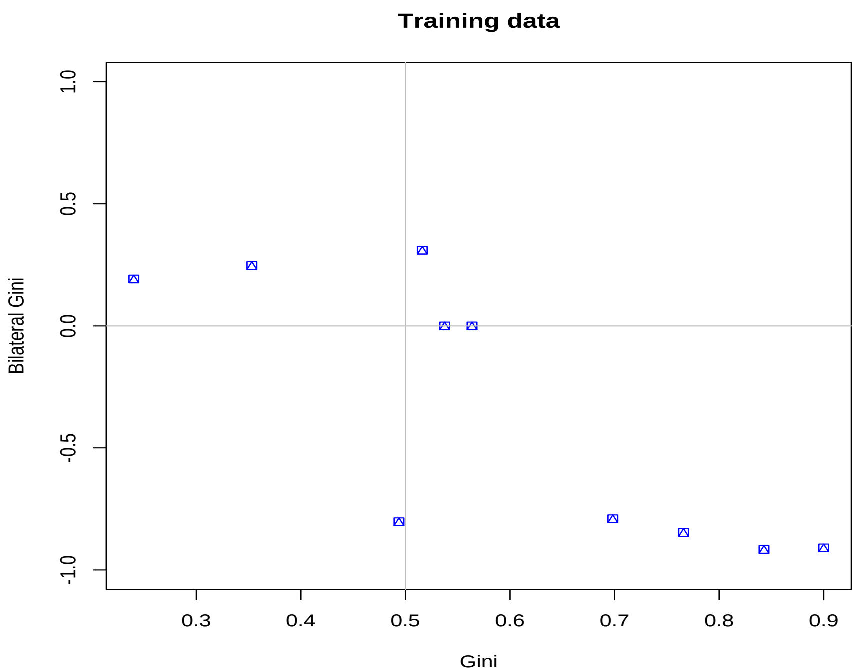
Finally, negatively inequality would be located in the economies of the poorest countries, where there would be a lot of inequality (Gini index values above 0.5), representing a great poor population base and a few with a lot of wealth, as it can be deduced, in this quadrant we would expect to find most of the economies of countries from Africa and Latin America with high unemployment rates, informality and job insecurity. However, as already explained, with an adjustment in the Gini values, this quadrant would surely be the one that best adjusts to the world economic reality.

Example. Let consider an illustrative example with $n = 100$, and $\bar{x}$=30.05. We consider 10 cases (subsets). Suppose we have $n_s = 10$ data in all cases. The data and results are the following:

Case	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	pM	pm	Gini	Bilateral
1	63	55	81	69	24	80	78	35	10	88	0.8	0.2	0.2403087	0.1922470
2	0	13	60	39	33	45	97	41	18	49	0.7	0.3	0.3531646	0.2472152
3	0	0	79	34	25	77	29	17	58	2	0.4	0.6	0.4937695	-0.8024922
4	0	0	0	38	100	61	44	53	99	3	0.6	0.4	0.5160804	0.3096482
5	0	0	0	0	23	98	87	37	68	71	0.5	0.5	0.5375000	0.0000000
6	0	0	0	0	0	93	48	59	82	86	0.5	0.5	0.5635870	0.0000000
7	0	0	0	0	0	0	56	57	94	20	0.3	0.7	0.6982379	-0.7905286
8	0	0	0	0	0	0	0	74	30	96	0.2	0.8	0.7660000	-0.8468000
9	0	0	0	0	0	0	0	0	70	28	0.1	0.9	0.8428571	-0.9157143
10	0	0	0	0	0	0	0	0	0	31	0.1	0.9	0.9000000	-0.9100000

In Figure 3 we plot the Gini vs Bilateral Gini indices obtained from the 10 simulated cases.

Figure 3. Bidimensional Bilateral Gini index obtained from simulated data.



Source: Authors' own creation.

As we can see in Figure 3, cases 1 and 2 have Gini indices less than 0.5 and $pM > pm$ (turbine 1). In case 3, the Gini index is less than 0.5 and $pM < pm$ resulting in a negative Bilateral index (turbine 2). Cases 4, 5, and 6 have Gini indices more than 0.5 and $pM \geq pm$ obtaining $BG \geq 0$ (turbine 3). Lastly, cases 7-9 have Gini indices more than 0.5 and $pM < pm$ obtaining $BG < 0$ (turbine 4).

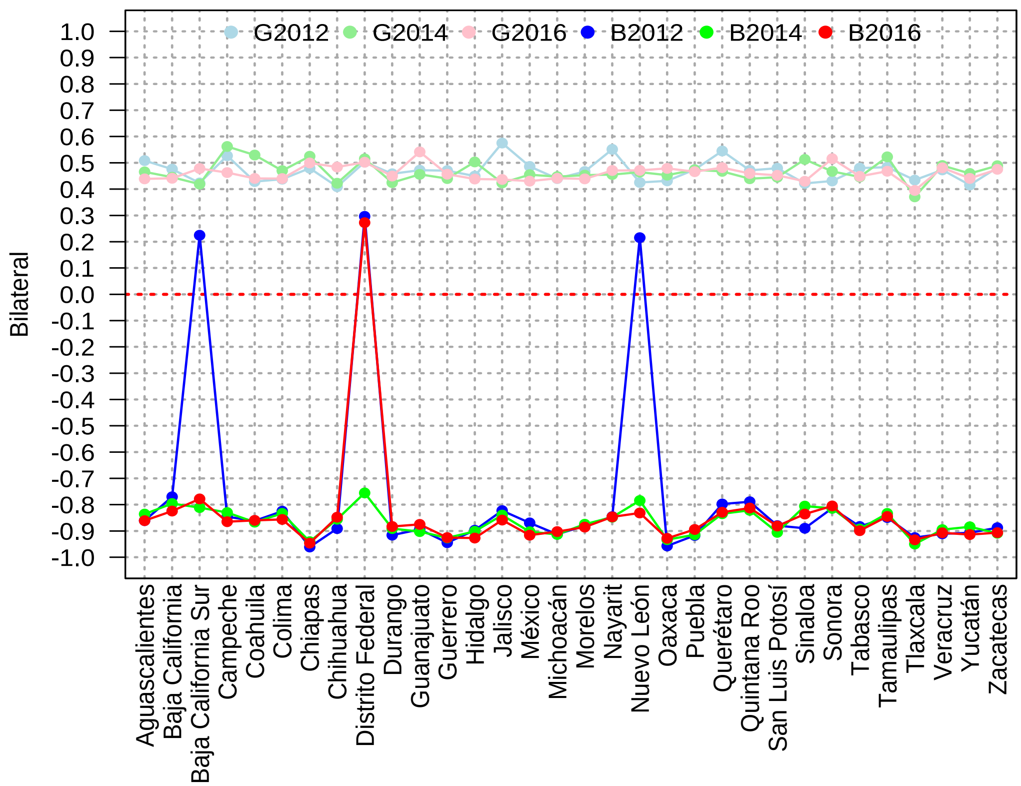
3. Application with real data

In this section we present two examples of calculating the Bilateral Gini index using real data. The first one analyses databases from Mexico, while the second one analyses international databases.

3.1. Bilateral Gini index from Mexico

Let consider the National Household Income and Expenditure Survey (ENIGH) (INEGI, 2018) from Mexico considering the years 2012, 2014, and 2016. We calculate the household income mean of Mexico for each year, and then the Gini index of each State (subsets of the population). Then, we calculate the Bilateral Gini index of each State. The results are presented in Figure 4.

Figure 4. Gini and Bilateral indices of the States of Mexico: 2012, 2014, and 2016.



Source: Authors' own creation.

The Gini indices for the States in 2012 are in the interval (0.409/0.575), in 2014 are in (0.371/0.562), and in 2016 are in (0.395/0.541). Once calculating the Bilateral Gini indices, we can see differences between the States. Indeed, we can also see these differences in the following analysis of classification.

Classification

In order to classify the States from Mexico in 7 groups (the number of groups is based on the economy regions of Mexico given by the National Institute of Statistics and Geography – INEGI), let consider a hierarchical cluster analysis with a complete distance method (James, Witten, Hastie, & Tibshirani, 2017). The results are presented in Figure 5.

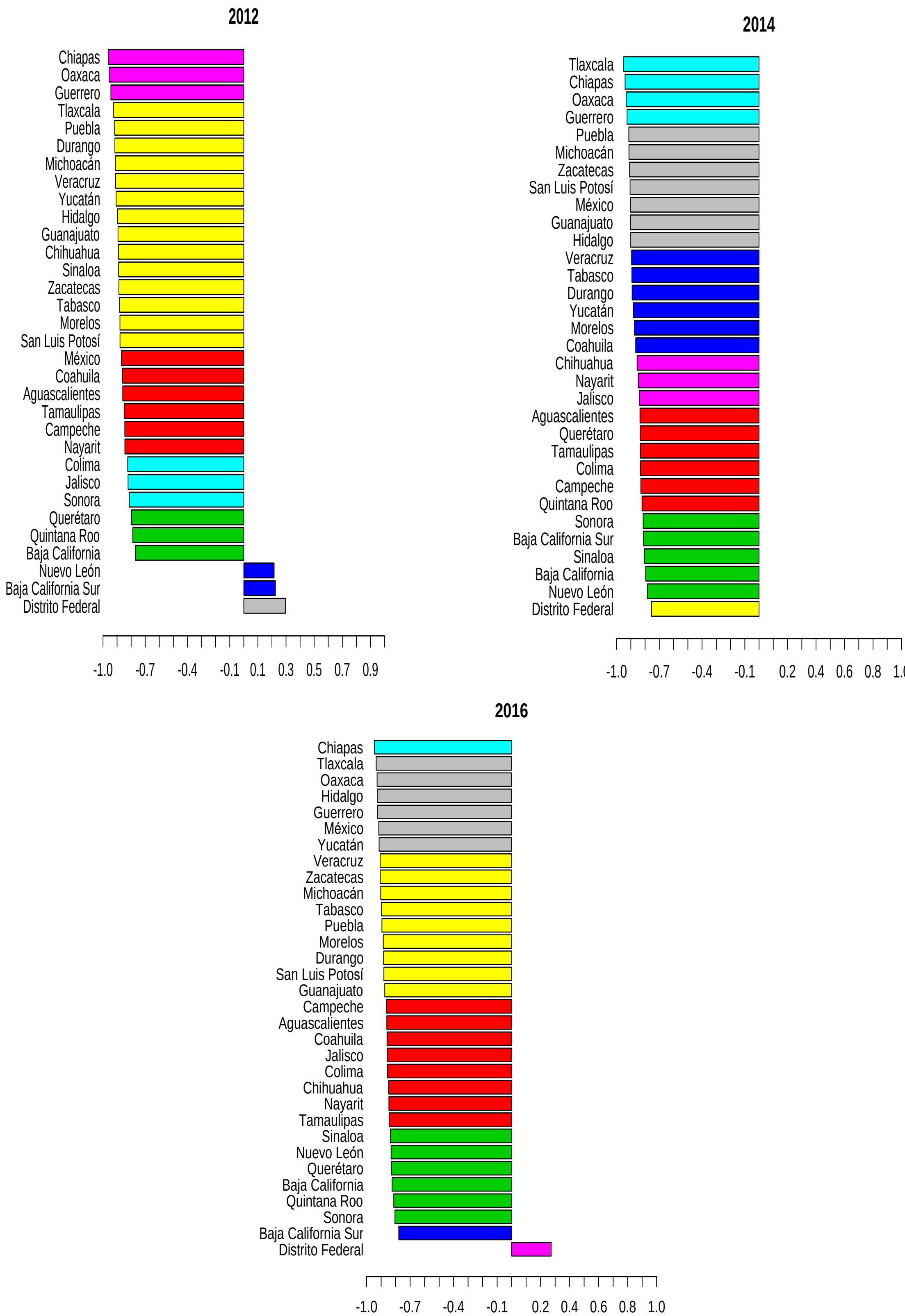
As most of the incomes above the average are concentrated in the Distrito Federal (now called Ciudad de México), compared to the other states, it implies a positive Bilateral Gini index (because the number of individuals earning more than the average is less than those earning below the average), however, when putting in the cost of living, use of time, and other indicators, it is certain that its classification will be different with respect to other States of Mexico. Moreover, given that in Mexico City there are people with incomes above the national average, taking the income as the only contextualizing variable, it shows the enormous inequality that exists in this city, classifying itself as a positive inequality state.

Notice that Baja California, Baja California Sur, and Nuevo León are border states, which have an important industrial development, which makes their conditions better than the rest of the country. Quintana Roo is the most important tourist center in Mexico, that is why it has significant investments in the tourism sector, in addition to a small population. In the other hand Chiapas, Oaxaca, Guerrero, Veracruz, Hidalgo, Michoacán, Guanajuato, Puebla and Zacatecas are in the group of the 10 states with the lowest Human Development Index (HDI) (PNUD, 2015), showing a coincidence of our results with the rank given by the HDI, which is built from three dimensions: income, health and education.

As we can see in Figure 5, the Bilateral Gini methodology was capable to capture the condition using the number of persons in poverty condition as an input. Note also that the states of Nuevo León, Baja California, Baja California Sur, Sonora, Sinaloa, and the Distrito Federal have had the highest HDI in last years, which also coincide with the highest Bilateral Gini, including Quintana Roo.

Zamudio-Sánchez, Santibáñez-Cortés, Viana-Carrillo, Andrade-Barrera, Jiménez-Machorro, Rodríguez-Esparza y Ávalos-Vargas (2017) have shown that the lowest human development index with services (that sets aside income and incorporates access to basic services) also coincide with our results which are the states of Oaxaca, Chiapas and Guerrero. The Bilateral Gini index shows very important information on the poorest states of Mexico, that have a significant indigenous population.

Figure 5. Classification of the States in México according to the Bilateral Index for 2012, 2014, and 2016.



Source: Authors' own creation.

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In Table 3 we present the percentage of the States in Mexico in each turbine of the Bilateral Gini index considering the years 2012, 2014, and 2016.

Table 3. Percentage of the States in Mexico in each turbine of the Bilateral Gini index.

Turbines	2012	2014	2016
1	6.250	0.0	0.0
2	75.0	78.125	90.625
3	3.125	0.0	3.125
4	15.625	21.875	6.250

Source: Authors' own elaboration.

Table 3 shows that the majority of the States of Mexico are in the turbine 2, which corresponds to equality in poverty, and it shows a country that over the years has increased the number of poor people (CONEVAL, 2015). In turn, the higher percentage in turbine 2 is a consequence of the parameter chosen in the acceptable Gini level, if this were defined by a lower value, the States would surely be distributed either in turbines 2 or 4, in which we would have poor States and rich States with poorly distributed income.

It should be noted that in turbine 3 appears a percentage greater than zero, due to the variable considered was income, and as mentioned in the methodology, the average income for the entire country was calculated. Taking this quantity as based, we calculate both the proportions of high and low income in each State; Mexico City, in some years, turns out to have a Gini index greater than 0.5, whose proportion of rich people is greater than the poor, with respect to the entire country, belonging to the turbine 3.

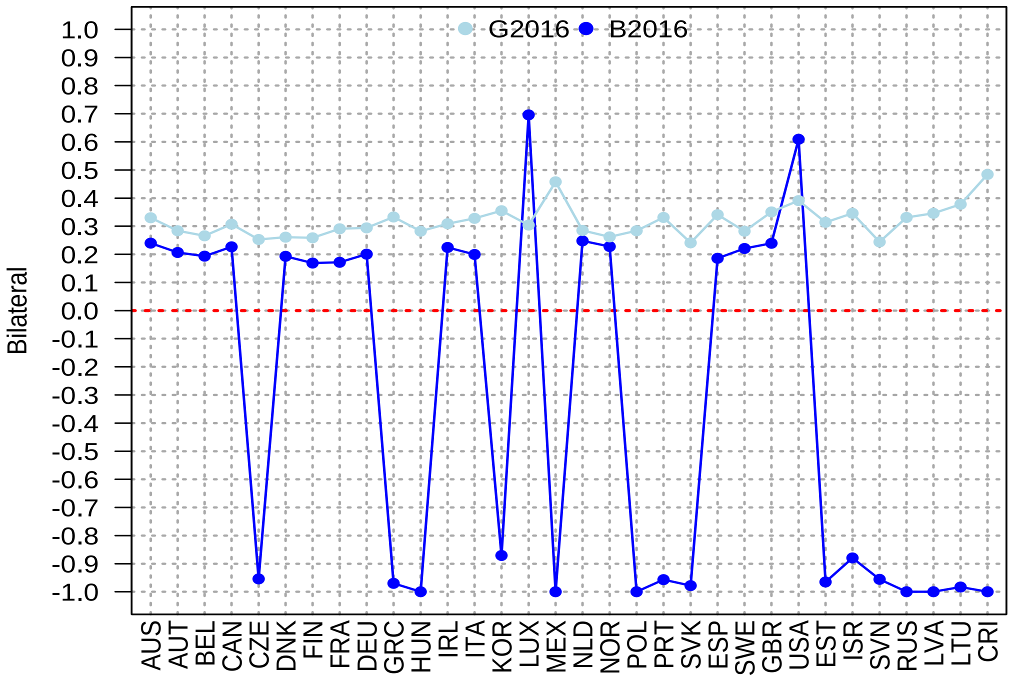
3.2. Comparison of countries

Now, we will consider an example considering different countries. We took the Gross National Income (GNI) Data (OECD, 2020a) of some countries from different years. We consider the Gini index of 2016 from the Income Inequality database of the OECD (2020b) of the countries, and we calculate their Bilateral Gini index. The mean of the GNI per capita from the countries was US$ 29508.51.

The abbreviation of the countries are as follows:

AUS Australia	DEU Germany	NLD Netherlands	USA United States
AUT Austria	GRC Greece	NOR Norway	EST Estonia
BEL Belgium	HUN Hungary	POL Poland	ISR Israel
CAN Canada	IRL Ireland	PRT Portugal	SVN Slovenia
CZE Czech Republic	ITA Italy	SVK Slovak Republic	RUS Russia
DNK Denmark	KOR Korea	ESP Spain	LVA Latvia
FIN Finland	LUX Luxembourg	SWE Sweden	LTU Lithuania
FRA France	MEX Mexico	BGR Bulgaria	CRI Costa Rica

Figure 6. Gini and Bilateral Gini indices of some countries, 2016.



Source: Authors' own creation.

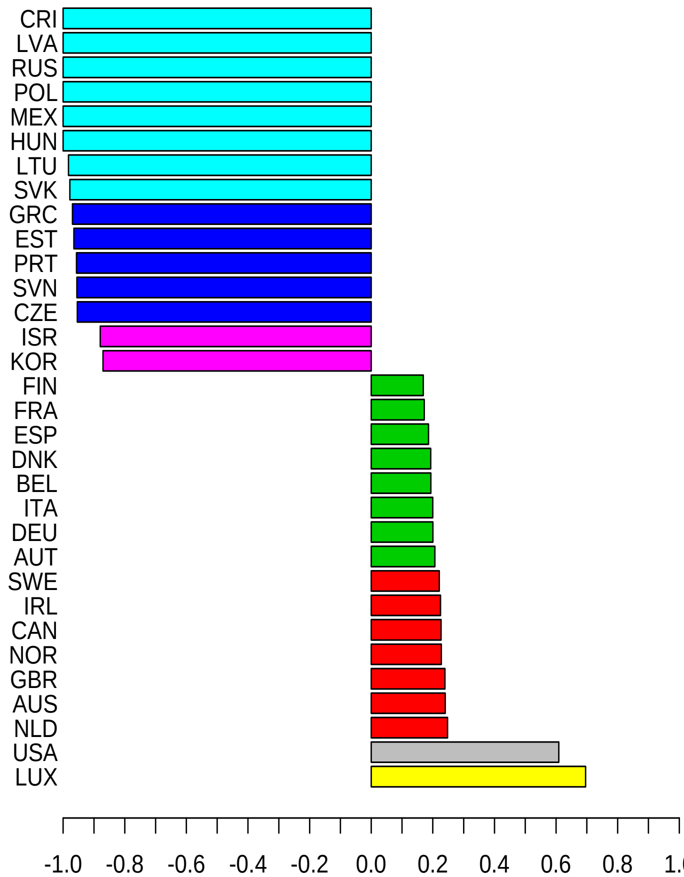
The results of the Bilateral Gini index of 2016 are presented in Figure 6. We could also classify the countries according to their Bilateral index into groups. The results are presented in Figure 7. The number of groups is arbitrary, in this case we classify into 7 groups.

Luxembourg and USA have $pM = 1$ but the Gini's of both countries are 0.304 and 0.391 respectively. Both countries have the best Bilateral results.

Note that countries like Finland, France, Denmark, Sweden, Canada, and Australia have obtained positive Bilateral Gini index, and are classified into the first turbine (positive equality). And for example, Mexico, Poland, and Hungary, have obtained negative Bilateral Gini index but they have Gini indices less than 0.5, so these countries have been classified into the second turbine (negatively equality).

UNDP (2019) shows the HDI for all the countries. In order to relate the results of the Bilateral Gini with the HDI, we see that countries with the lowest Bilateral index belong to the 10 HDI lowest HDI in the European Union. The same case is happening with the highest Bilateral index and the highest HDI. That is, we have shown a positive correlation between the Bilateral Gini index and the HDI. Nevertheless, having a Gini index, for example, of 0.4 in a Nordic country it is not the same (has a different meaning) as having this inequality index in an emerging country.

Figure 7. Countries classification via the Bilateral Gini index. 2016



Source: Authors' own creation.

Table 4 presents the percentage of the countries in each turbine of the Bilateral Gini index considering the year 2016. In this table, we can see a considerable number of countries being in the turbine 1, i.e., positive equality. It should be pointed out that the Gini values go from 0.241 to 0.484.

Table 4. Percentage of the countries in each turbine of the Bilateral Gini index considering the year 2016.

Turbines	%
1	53.125
2	46.875
3	0.000
4	0.000

Source: Authors' own elaboration.

Mexico is classified in the turbine 2 (negative equality), and in Table 3 we see that more than 90% of the states of this country in 2016 are classified also in this turbine, showing the robustness of the new index.

Conclusions

In this article we have shown an alternative analysis of the Gini index. This index *per se* is an indicator of inequality, but it does not allow a "fair" comparison between countries or territories since there are factors that may vary between regions that give a particular context. In that sense, the proposed methodology reinforces and make a more complex analysis of the Gini index giving elements to infer economic and social conditions of a given Gini index value.

The Gini index is an invaluable input to understand the changes in the social structure of countries, however, by itself, due to the way it is constructed, it is not enough to give a deep explanation about the conditions, social movements, impact of public policies, social phenomena and other aspects that should be reflected in a measure of inequality. In addition, the ideological context that it configures must be considered.

The proposed analysis avoids, in some way, giving an erroneous interpretation of the variations (decrease or increase) in the value of the Gini index, providing a more realistic context through the visualization over quadrants. These quadrants represent turbines that can be loaded with specific and relevant information. In this article, we only used the income as an input, however, the turbines can incorporate other analysis factors to land the quadrants in a social, historical, political context, among others, across economies.

Therefore, in the example of the Gini index value of 0.4 that we mentioned in the first part of this paper about the countries of El Salvador and the United States, with the presented methodology, it is clear that, although both countries have the same Gini value, the context in which this inequality occurs is differentiated, since the inequality registered in the quadrants gives us a reference about the conditions in which the Gini value is given, which also allows us to understand the meaning and implications in the variation of the measure of inequality.

The results of the calculation of the Bilateral Gini index for Mexico show the differences of regions within the country: the states that despite having better income have greater inequality, or failing that, states with low incomes in a condition of equality (equity in poverty). Due to the visualization of differentiated conditions within the country, we require new strategies to discover the path that economies must travel to achieve a fairer and more equitable income distribution; thus, the Bilateral Gini methodology can help public policies makers more effective interventions to combat inequality.

In general, as the construction of the Bilateral index is given, it depends on the Gini index, but, the Bilateral index considers more variables of the studied population by adding context through the estructural differences in the deciles composition. We can corroborate that feature in both cases: analyzing the states of Mexico and through the analysis of countries.

According to Cortés (2017), data showed that in Mexico, although there was a reduction in the value of the Gini coefficient in the 1984-2014 period, the share percentage of income of the decile X increased, which implies a growth of the population living in poverty; in addition, the population within deciles IX and X concentrates 50% of the Mexican population, this helps us to visualize what the Bilateral Gini tries to contribute to the Gini Index: the decrease in the Gini value does not imply an improvement in the average income of the population. A clear example is the unusual decrease in inequality in the State of Tlaxcala (CONEVAL, 2019), one of the poorest states of Mexico, which according to official data in 2015 had the lowest Gini value of all the states of the country (0.411), however, according to our methodology, it could not be considered a state with an improvement in its income (or in its living conditions) since Tlaxcala have one of the highest percentages of people living in poverty (58.9%). In this aspect, we assume, that this is an important information to consider in the evaluation of inequality.

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