The Real as a Topological Manifold

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Abstract

We re-examine a discussion of "idea and multiplicity" (as a manifold) as presented by Gilles Deleuze in his work *Difference and Repetition* and develop this topic according to the philosophical concepts of *support, real* and *reality* introduced by the first author. We characterize in a precise way the topological aspect of the Idea and extend this comprehension to the real.

1 Introduction

In this article we propose to investigate and extend the relations suggested by Gilles Deleuze concerning the concepts of Idea and Riemannian manifold [1, 2][1]. We will base our arguments on the concepts of *support*, *real* and *reality* introduced by A. Martins [3] and we will consider, as a starting point, the philosophies of Spinoza and Gilles Deleuze [2] by using the Deleuzian concepts of *virtual (or potential) and actual* and the Spinozian concept of *mode* [6]. We will begin by giving a brief explanation of the concepts we will use [3]. The *support* (Martins [3]) describes, indicates and gives a comprehension of an object, although the object does not reduce itself to the support that describes it. The support is not identical to the real object but it is a statement, a form, a *mode* [6] or even a content, but never the whole real object that simply *is* - it is in movement, in changing, in *devenir*; it does not let itself be adhered to the support and it does not crystallize itself, therefore, it does not allow a description of the object "in-itself". However, our perception of the object or our description of it does not correspond to its *manifestation*, but indicates its *expression*: the real object does not manifest itself in a phenomenon (different from the thing or the real object in itself) but it is expressed in the relation in which we take part, a relation of perception, interpretation, statement.

The *real* and the *reality* [3] are concepts we employ to describe two aspects of a same present real (or of a same real object): the reality, as it is assumed in the usual sense, means effectivity, effective experience or, simply, the elements of the world; as it is found in dicctionnaries, the reality also means: the quality of being actual, that which exists objectively, and, in fact, the set of things possessing actuality. But the idea, or now the concept of reality, doesn't explain the genesis of their elements, i.e. the implicit condition of the things, events or occurences. The real is a concept that gives understanding to this plane or ground where the effectivity emerges (Deleuze says it is a virtual plane, which is real but not actual and gives origin to the actual things). Thus, we can say that the reality actualizes the real, or the reality is the actual and local version of the real. There is no reality without real, or also, there is no reality that is not an expression of the real, as well as there is no real that does not constitute itself in some reality. In other words, the real - or according to Spinoza, the *Substance* doesn't *exist* in a pure form but as undergoing modification under the form of mode: the reality is a mode of the real [4]. In Deleuzian terms, the real is a present *virtuality* that becomes *actualized* as reality, and like a real object, the real is actualized in its relations by supports. The real object, considered previously to its supports, namely without its actualizations, is virtual, potential. But, this potentiality (that is not a possibility [5])

[1] Chapter 4, Idea and the Synthesis of Difference, pp. 182-184.

[2] Deleuze approaches his own philosophy to Spinoza's philosophy through two of his books, *Spinoza et le problème de l'expression* [4] and *Spinoza: philosophie pratique* [5].

[3] All citations of Gilles Deleuze's book [2] will be referred in quotation marks. We also take the initiative to rewrite some parts according to the original meaning of the French text [1].

[4] The modes are modifications of (and into) the substance (Ethics I, def.5, cf. [6]). Therefore, the substance is imanent to its modes. The substance (*Naturam naturantem*) *exists* as mode (*Naturam naturatam*), which can be infinite - if they arise from the infinite nature of one of the substance's atributes, e.g. the infinite intellect, the infinite will, the modes of thinking, of motion and rest, the modes of extension), or finite - if they arise from the finite nature of one of the substance's attributes, e.g. the individual things, the finite intellect, the finite will, the finite motion or rest. The substance, through their attributes, is expressed by the finite modes (Ethics I, Prop. 21 to 32, cf. [6]). Then, we can say that the reality (*Naturam naturatam*) is the actual mode of the real (*Naturam naturantem*).

[5] Cf. G. Deleuze [2], Chapter 4, "The distinction between the virtual and the possible", pp. 211, 212.

is present and constitutes the actual object, the actuality of the real object, and there is no real object that does not have actuality. Virtual (or potential) and actual are two aspects of a same real object.

2 Topological Manifold

By a n-dimensional *topological manifold* we understand [7] a non-null set X together with an atlas $\{(\phi_i, \mathcal{U}_i), i \in I\}$[6] where $\{\mathcal{U}_i, i \in I\}$ is an open covering of X i.e., $X \equiv \cup_{i\in I} \mathcal{U}_i$ and ϕ is a homeomorphism of $\mathcal{U}_i$ into an open set of R^n (each pair $(\phi_i, \mathcal{U}_i)$ is called a chart of X) satisfying the following conditions:

(i) For all two charts $(\mathcal{U}_i, \phi_i)$ and $(\mathcal{U}_j, \phi_j)$ such that $\mathcal{U}_i \cap \mathcal{U}_j \neq \emptyset$ we have that $\phi_j \circ \phi_i^{-1} : \phi_i(\mathcal{U}_i \cap \mathcal{U}_j) \longrightarrow \phi_j(\mathcal{U}_i \cap \mathcal{U}_j)$ is a homeomorphism between the open sets $\phi_i(\mathcal{U}_i \cap \mathcal{U}_j)$ and $\phi_j(\mathcal{U}_i \cap \mathcal{U}_j)$ of R^n (a continuous inversible map whose inverse is also continuous).

(ii) The atlas is maximal concerning to property (i), i.e. given a chart $(\mathcal{U}, \phi)$ of X such that for all $i \in I$ with $\mathcal{U} \cap \mathcal{U}_i \neq \emptyset$ the map $\phi \circ \phi_i^{-1} : \phi_i(\mathcal{U} \cap \mathcal{U}_i) \longrightarrow \phi(\mathcal{U} \cap \mathcal{U}_i)$ is a homeomorphism between open sets of R^n, then the chart $(\mathcal{U}, \phi)$ also belongs to the atlas of X.

This formal definition needs some explanation in order to be related to our concepts. A manifold is a set X together with a specification of a family of subsets $\mathcal{U}_i' s$ of X that covers it, i.e. the union of those sets $\mathcal{U}_i' s$ results in the set X. We call the set $\{\mathcal{U}_i, i \in I\}$ a covering of X. Of course, we can assign other families of subsets $\mathcal{U}_i' s$ that also covers X, so that a specific covering corresponds to an *outline* [3] of X. Any set can be considered as a union of its subsets (despite the specific choice of subsets, or outline, we use). A set is then a whole covered by its parts, the subsets that cover it. Those subsets compose the whole again by the relation of union of its parts, however, any other choice of covering will not modify the whole. This encodes the fact that the whole foregoes its parts, but it doesn't do without *a* specific part since the set is only conceived with a certain covering (even in the trivial example of taking a covering as being the whole set, which implies that the whole is also a part).

3 The Idea as a Topological Manifold

Now, following Deleuze's text [2], we will investigate the characterization of Idea as a topological manifold according to our concepts. We start from his definition: "Ideas are multiplicities: every idea is a multiplicity or a manifold" (Deleuze remarks that this is a "Riemannian usage of the word 'multiplicity' ").

(A) The Idea as a topological manifold: To Deleuze, "An Idea is a n-dimensional, continuous, defined multiplicity".

(A.1) *By defined multiplicity:* we understand the elements of the Idea. The Idea, being a manifold (that we will also refer as multiplicity), is a non-null set and, therefore, it contains *elements* that we will refer from now on as points. On a manifold, any point p is described by n-coordinates (n being the dimension of the manifold) that are specified by a chart $(\mathcal{U}_i, \phi_i)$ that contains p ($p \in \mathcal{U}_i$). This is done through the homeomorphism ϕ_i that takes p into a point $\phi_i(p)$ of R^n specifying then the n-coordinates of p. This chart is said to define a (local) coordinate system on $\mathcal{U}_i$, a neighborhood of p. The choice of a specific coordinate system on a neighborhood of a point corresponds to a support used to describe the point (we call such a process a "coordinatization" [7]). There are many different coordinate systems that provide a description of the point, all of them equivalent in the sense of being reciprocally determined as we will explain in the sequence, and this means that there are several supports. The equivalence among all these coordinate systems describing the *same* point tell us that coordinate systems, as a support, neither add nor take take away anything from the point. It asserts that there is not an identity and what is defined is not the point (the mode) but each support that indicates it.

Using Spinozian concepts, the point is understood as a mode, or in Deleuzian terms as a virtuality or potentiality, i.e. as pre-coordinatization, previously to the support that defines the point. The coordinatization defines the point by the support adopted, but it does not determine the point as an identity, for the point is not identical to its n-coordinates, otherwise it would not admit more than one coordinatization. The point, as virtual, does not assume any coordinatization, but it can only be described by coordinates, and it actualizes itself by its own description: it actualizes in a support. We can say that the *real* point, as a mode of the real, only exists in a specific way and it expresses itself through some support as reality (as a specific reality).

(A.2) *By dimensions:* we understand the number of variables, namely the number of coordinates necessary to describe the points of the manifold. Note that, once the number of dimensions is specified, we can then define a coordinatization for the points of the manifold by choosing a support.

[6] We will denote R the set of real numbers. I is the set of indices i which labels the open sets $\mathcal{U}_i$.
[7] Even though this word seems not to exist in the current English language.

Any manifold carries its dimension as an intrisic characteristic that specifies and delimits itself to be an n-dimensional topological manifold. There are other topological properties such as compactness, conectedness etc, that when considered delimit even more the manifolds by assigning to them topological invariants. This delimitation does not imply an identity, as in a kind of platonic manifold. It only determines an equivalence of the manifolds as being n-dimensional.

(A.3) By continuity: Deleuze wrote, "By continuity, we mean the set of relations [8], between changes in these variables". The point is defined by means of its coordinatizations, each one associating to the point a set of n-uplas, and the relations between one coordinatization and another, (i.e. between one set of n-uplas and another) are well defined in the sense of being a homeomorphism that means, in mathematical terminology, a continuous and open map. Given a chart $(\mathcal{U}_i, \phi_i)$ of the manifold X, if we take two closer points $p, q \in \mathcal{U}_i$, the continuity of ϕ_i will associate the corresponding images $\phi_i(p), \phi_i(q) \in R^n$ also as closer points. However, the fact that ϕ_i is a homeomorphism tell us that not only ϕ_i is continuous but also that there is a continuous map ϕ_i^{-1}, the inverse of ϕ_i. The existence and continuity of ϕ_i^{-1} will be necessary in order to guarantee the equivalence of different coordinatizations of a given point, i.e. the reciprocity of the different coordinatizations of the point. This issue will be analyzed further (see B.2).

(A.4) We conclude our analysis stating: the idea is a manifold, i.e. a set of elements, each of them being described by a set of n coordinates (multiplicity with n-dimensions). Any element may admit several coordinatizations, which describe the element without hierarchy. Each coordinatization is related to another through a homeomorphism. The Idea is constituted by elements that are described through supports. The distinct supports are equivalent in the sense of providing an equivalent description of the Idea.

Let us now turn to the question of the "moment at which an Idea emerges".

(B) There are three steps concerning the emergence of the Idea.

(B.1) "The elements of the multiplicity must have neither sensible form nor conceptual signification, nor, therefore, any assignable function. They are not even actually existent, but inseparable from a potentiality or a virtuality." The point destituted of an actual existence is indeed the pre-coordinatized point because the actual existence of the point (i.e. the actualization of it in a description) corresponds to a coordinatization. Then, as pre-coordinatization, the point is virtual and has a potential (individuated as point) of actualization [9]. It is in this aspect that we say the points "imply no prior identity" with a coordinatization, with "something that could be called one or the same" or as if the point was identical to some coordinatization. The non-determination of the point makes it possible the manifestation of the difference released from every subordination: non-determination is a characteristic of the point when understood as pre-coordinatization (it is not determined in itself but only in its coordinatizations). The manifestation of the difference becomes evident in the different coordinatizations the point admits. On the other hand, the non-subordination becomes evident for there is no hierarchy among the coordinatizations due to the fact that none of them are necessary (neither *a priori* nor *a posteriori* since it is only a support and it does not constitute an identity or nature of the point).

(B.2) "These elements must in effect be determined, but reciprocally, by reciprocal relations which allow no independence whatsoever to subsist." What determines the point is not a specific coordinatization, but the fact that its coordinatizations, being related by homeomorphisms and consequently equivalent, refer to the same point. Then, they repeat it differentially, since each coordinatization is different from the other. The dependence of the coordinatizations mentioned by Deleuze is the result of these equivalences or reciprocity. If these coordinatizations were not equivalent, they would either indicate different points or they would assign the existence of a more adequate coordinatization, constituting then hierarchical supports that would not indicate the same object.

The difference actualizes itself on the several definitions that repeat the point. In this way, the points are determined, but not as pre-coordinatization. It is the indetermination of its pre-coordinatization that makes homeomorphic definitions for the points possible. If the point was determined as pre-coordinatization, then there could only be one coordinatization corresponding to this pre-determination, establishing an identity. Therefore, this coordinatization would be the only one possible or true, admitting a homeomorphism only to itself. The "identity" of the point is given neither by a previous identity (as a pre-coordinatization), nor by an a posteriori identity (by a supposedly more adequate coor-

[8] In the original French text [1], Deleuze uses *rapport* meaning the virtual and *relation* meaning the actual. We will translate the former by relation and the latter by correlation. The English translation (1994) does not observe this important conceptual difference, using relation and relationship indistinctly, which justifies our initiative to modify it.

[9] The pre-coordinatized point exists as potentiality. This is equivalent to say that a point of a manifold, seen purely as an element of a set, is understood philosophically as having a potential existence. The point when seen through its homeomorphic image in R^n becomes the actual or real point.

dinatization), but through the repetition of its many related differences (all its equivalent coordinatizations). The non-determination does not correspondto a bad definition, but indeed to a non-previous-identity.

"Such relations are precisely non-localizable ideal connections, whether they characterize the multiplicity globally or proceed by the juxtaposition of neighboring regions". Any point p is considered as belonging to an open subset (i.e. an open neighborhood), $\mathcal{U}$, of the manifold. Let us consider that the point p belongs to more than one open subset, $\mathcal{U}_1, \mathcal{U}_2, ..., \mathcal{U}_i, ...$ Each $\mathcal{U}_i$ is associated with its image $\phi_i(\mathcal{U}_i) \subset \mathcal{R}^n$ by a homeomorphism ϕ_i that parametrizes it in a continuous, univocal and reciprocal way. The intersection (juxtaposition) of neighborhoods, $\cap_{i \in I} \mathcal{U}_i$, of the manifold is mapped into different regions $\phi_i(\cap_i \mathcal{U}_i) \subset \phi_i(\mathcal{U}_i)$ of $\mathcal{R}^n$. Although being distinct regions of $\mathcal{R}^n$ they are the same region in the manifold, so that they should be related in $\mathcal{R}^n$ by homeomorphisms.

These relations are ideals, conceived as an abstraction, since the related regions are images in $\mathcal{R}^n$ of a same and unique region in the manifold, i.e. these relations only exist in the description of the intersection of the several $\mathcal{U}_i's$ on $\mathcal{R}^n$. They are non-localizable in the sense that the point does not belong to a specific region $\phi_i(U_i)$ of $\mathcal{R}^n$, but belongs to all of them. The relations do not exist as an element of the manifold, i.e. a point, as it would seem by the use of the term 'non-localizable'; they are just functions that appear relating the $\phi_i(U_i)$ of $\mathcal{R}^n$.

In the particular case where the whole n-manifold is homeomorphic to R^n (and in this case homeomorphic to any open ball of R^n) we can describe $\mathcal{M}$ by means of a unique chart $(\mathcal{M}, \phi)$ that maps the whole manifold into a subset $\phi(\mathcal{M}) \subset \mathcal{R}^n$. We can also, possibly, use another chart, $(\mathcal{M}, \chi)$, that maps $\mathcal{M}$ into another subset $\chi(\mathcal{M}) \subset \mathcal{R}^n$. These are, in general, different regions of $\mathcal{R}^n$ that are related through the homeomorphism e.g. $\phi \circ \chi^{-1} : \chi(\mathcal{M}) \rightarrow \phi(\mathcal{M})$. In this particular case, as in the juxtaposition of neighborhoods describe previously, the relations among the regions on $\mathcal{R}^n$ are also ideals and non-localizable.

"In all cases the multiplicity is intrinsically defined, without external reference or recourse to a uniform space in which it would be submerged." The manifold is a reference to itself, it does not demand to be referred to another space. In other terms, the multiplicity is not defined by an identity, nor from a third term, external to the individual or to the system that presents itself as a model for them; the individuation of multiplicity is fulfilled by a differential repetition, by its intrinsic characteristics.

"Spatio-temporal correlations no doubt retain the multiplicity, but lose interiority". The coordinatizations *indicate*, on R^n, the multiplicity, i.e. they 'retain' the multiplicity when describing it. Nonetheless, they lose interiority, i.e. the characteristic of the real point, since the real point is not conceived as pre-coordinatization any longer. It is not even possible to *define* the real point in itself, for doing that necessarily makes use of a support, whether it corresponds to a coordinatization or to an axiomatic concept of the point.

"Concepts of the understanding retain interiority, but lose the multiplicity, which they replace by the identity of an 'I think' or something thought." The point, as a primitive (axiomatic) concept, is just an intellectual abstraction [10], a concept of the understanding, whether it is given by the conscience, by the *cogito* or by a phenomenology of the object that is thought. This concept *indicates* the interiority of the multiplicity due to a pretense *identity* of the axiomatic concept with the real point. Besides that, this concept 'retains' the interiority of the real point in the sense of requiring a comprehension of the latter. But losing multiplicity is a reduction that does not describe the characteristics of the multiplicity, for example a particular covering for the manifold or the diversity of coordinatizations.

This diversity of coordinatizations indicates also a diversity of/in the multiplicity, by the fact of existing several coordinatizations of the point, while the axiom assigns the unity of the point as being only one. The real point is, at once, one and many. It is one differentially by the repetition of its intrinsic difference.

"Internal multiplicity, by contrast, is characteristic of the Idea alone." The internal multiplicity is nothing but the multiplicity as pre-description, i.e. characterized by a certain covering without considering the images of their open sets on R^n. It offers an infinite potentiality of outlines understood as coverings or coordinatizations.

(B.3) "A multiple ideal connection, a differential *relation*", i.e. the juxtaposition of neighborhoods, the intersection of $\mathcal{U}_i's$ on the manifold, "must be actualized in diverse spatial-temporal *correlations*", i.e. in the homeomorphisms among the $\phi_i(\mathcal{U}_i's)$ that are the parametrizations on R^n of the juxtaposed regions. This, "at the same time as its *elements*", the points, "are actually incarnated in a variety of *terms* and forms", the coordinatizations of the point. The actualization of the juxtaposition of neighborhoods *on the manifold*, in the homeomorphics correlations *on* R^n corresponds to the actualization of the point by coordinatizations.

"The Idea is thus defined as a structure". The term structure can mislead since it induces us to believe on the existence of a fixed structure determining the singular actual particularities as understood by the Structuralism, e.g., from the linguistic to the Lacanian psychoanalysis. Therefore, if we intend to employ this term we can do it in the precise sense

[10] on the spinozian terms, a universal notion formed by the reason or the second gender (Ethics II, Prop. 40, Sc. 2, cf. [6])

established by Deleuze: a virtual structure that is global, non-structural or structured. It is but origin and genesis of the actual individuations. This virtual structure is immanent, constituting the individuations.

There are many levels concerning the description of the real. The first one corresponds to the real as a manifold given by its mathematical definition and then constituting a common virtual structure. This mathematical definition of the manifold is present and composes any of its ways of being without getting tangled with a specific way of being. It implies a generic property that is implicit on its definition. The manifold, as well as its characterization (as admitting a generic atlas, which constitutes the second and third levels of description of the real), only exists as a specific manifold, e.g. a sphere, a projective space, etc. The (mathematical) concept of manifold repeats itself differentially in the specific manifolds. In the first level the real is described as a general real, i.e. the manifold as a mathematical concept, or taken as an individuated real, i.e. as a specific manifold. We remark that the specific descriptions, e.g. as spheres, projective spaces and so on, take place in the *language*, in the formalism, in the abstraction, but all of them refer to spheres, projective spaces, etc. as a way of thinking the world. The second level of description of the real is given by the covering $\{\mathcal{U}_i, i \in I\}$ and the third level by the mappings $\phi_i : \mathcal{U}_i \rightarrow \phi_i(\mathcal{U}_i) \subset \mathcal{R}^n$.

This second and third levels of description of the real occur both for the real as a generic or a specific manifold. The second level corresponds to the reality while the third corresponds to a description of the reality, a statement. The third is a support that indicates the second, that by its turn indicates the first. All of them are supports for comprehension of the real and take place simultaneously since they point out intrinsic properties of the real that are implicit into its mathematical definition. The support is not a manifestation of the real, as it would be if the manifold were separated from its specific cases and manifested itself only on them; since the manifold constitutes entirely all of its specific cases we have that the support expresses the real. More than representing the real, the manifold while describing it does not replace the real but indicates it.

We emphasize that the mathematical structure of the manifold is only a support of comprehension of the real. That means, the manifold is not identical to the real. We are employing the concept of manifold as a support to the real because, since the manifold behaves like the real in its actualization as reality, it takes account on the proprierties of the real. On the contrary, if considered not just as a support of comprehension, we could think of a set that is not a manifold and consequently conclude that there is something beyond the real; that is not true.

Since the manifold constitutes its specifics cases, it is not metaphysically transcendent in respect to them or to the objects they describe; in the same way, it is not a perfect model of imperfect copies. There are no copies nor models, but only virtual structure and individuations as actualizations of this virtuality. If we conceive the relation of the manifold with its specific cases as the relation between the model and its copies (according to a non immanent view), then when we consider specific manifolds as homeomorphics, its homeomorphism would be explained from a *similarity* to an *external term*, the model [11]. Therefore the homeomorphism would exist by *analogy* to this model. The similarities among the forms proceed through the identification of each form to the model, the manifold, considered as an external agent, and not between them directly. But in this case, the different forms keep separated and identical to themselves. Nonetheless, considered as modifications into (and of) a same substance (according to Spinoza) or a virtual structure (Deleuze), the homeomorphisms are viewed as a transformation of a mode to another mode, in and of the substance, as in a same body or in a same tissue. There is only one "identity", that is however *intrinsically differential*: the substance. A unique being expressed on many singular modes, a *univocal* manifold that repeats differentially on its many cases or non-separable forms.

4 Final considerations

In this work we have shown how the philosophical concepts of real and virtual appear in the theory of topological manifolds. This suggests that philosophical concepts may extend their domain to disciplines that are frequently considered untouched by philosophical ideas. In fact, it is our belief that science has very much to gain with a philosophical inquiry of its fundamental concepts.

The main intention of this transdisciplinary study is not only of establishing a parallel between concepts of philosophy and other disciplines, but also of using this parallel as a bridge that may allow us to use our philosophical insights into the development and reinterpretation of concepts coming from other areas.

In Physics, for example, the concepts of real and the pair virtual/actual appear when we treat the concept of wave function. Specifically, from the principle of superposition of states we know that the wave function of a certain system can be in a state defined by a sum of eigenstates of a complete

[11] Of course there are many classes of homeomorphic manifolds such that manifolds belonging to different classes are not homeomorphics. Each class, that we can consider as a genus or a specie, is a mode of the manifold as a mathematical concept that constitute them. Then, in spite of not being related by a homeomorphism the univocity principle is still valid: the different genus or species are modes of a same substance or virtual structure.

set of commuting observables. In this stage, we say that the wave function is virtual. After a measurement is performed, the wave function is reduced to one eigenstate and we say that the wave function is actual [12]. The measurement corresponds then to the process of actualization of virtuality into reality. In the case discusssed in this work, the actualization of the pre-coordinatized point (virtual) into its image in R^n (actual) corresponds to a homeomorphism. It is interesting to see how different objects behave in a similar way when seen from the same philosophical perspective. In Physics, the concepts of real and the pair virtual/actual appear, for example, when we treat the concept of wave function. Specifically, from the principle of superposition of states we know that the wave function of a certain system can be in a state defined by a sum of eigenstates of a complete set of commuting observables. In this stage we say that the wave function is virtual. After a measurement is performed, the wave function is reduced to one eigenstate and we say that the wave function is actual. [13] The measurement corresponds then to the process of actualization of virtuality into reality. In the case discusssed in this work, the actualization of the pre-coordenatized point (virtual) into its image in R^n (actual) corresponds to a homeomorphism. It is interesting to see how different objects behave in a similar way when seen from the same philosophical perspective.

Here, it is important to notice that the wave function pre-measurement, and the pre-coordenatized point are objects with *distinct* nature that, nonetheless, exhibit the *same* virtuality. The process of actualization of the wave function, which is determined by a measurement, also differs from the process of actualization of the point, which is determined by a local homeomorphism. In both cases, we have identified the *same* concept virtual/actual that, due to the *distinct* nature of the objects under consideration, is *expressed* in different ways. A detailed study of the concepts of real and the pair virtual/actual in quantum mechanics are being investigated by the authors and will be presented elsewhere.

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[12] Of course, the word actual is being used in a philosophical sense. We are not implying that after measurement the wave function becomes a physical entity that can be measured.

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