Physicæ

A simple description of the interaction between photons and quasi-deuterons

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The relativistic energy-momentum conservation principle and the Pauli Exclusion principle have been applied to a simple description of the interaction between photons and neutron-proton pairs (quasi-deuteron) in photo-induced nuclear reactions in the energy range ∼ 30-140 MeV. As examples, the kinetic energies of the neutron and proton after the primary photo-interaction have been calculated to ^{27}Al and ^{238}U nuclei as a function of the proton polar angle in the center-of-mass system for incident photon energies of 50 and 100 MeV.

Keywords: Photonuclear reactions, Photon absorption and scattering

I. INTRODUCTION

The photon-nucleus reactions are usually classified according to the wavelength (or energy) of the incident photon. Roos and Peterson [1], based on data accumulated in the early years of experimental investigation of photonuclear reactions, have grouped such reactions in three different energy regions as shown in Fig. 1. For energies up to about 30 MeV the photon wavelength is of the order of magnitude of the nuclear diameter (∼ 10 fm) and, therefore, the incident photon interaction with the nucleus as whole. From about 30 MeV up to 140 MeV the wavelength of the incident photon is of the order of magnitude of structures formed by a neutron and a proton (∼ 3 fm), and the interaction takes place with these quasi-deuterons. Finally, above about 140 MeV the primary photointeraction occurs with individual nucleons, since the photon wavelength is of the order of magnitude of the dimension of a free nucleon (∼ 1.5 fm). The first region, up to about 30 MeV, is known as giant resonance region, and it is characterized by a strong interaction of the incident photon with the dipole moment of the nucleus as a whole; the region between the end of giant resonance and ∼ 140 MeV (the energy threshold for π-meson photoproduction, is called the quasi-deuteron region; from ∼ 140 MeV on we have the so-called photomesonic region.

The nuclear fission reactions induced by photons in the quasi-deuteron photoabsorption energy range (∼ 30–140 MeV) have been currently studied on the basis of a two-step model. During the first stage of the reaction it is supposed that the incoming photon is absorbed by the target nucleus via the interaction with a neutron-proton pair (quasi-deuteron). This mechanism was described by Levinger's model of photo-interaction with quasi-deuterons (firstly proposed in 1951 [2], and later modified in 1979 [3]), where the incident photon energy is distributed between these two nucleons. Soon after the primary interaction takes place, and depending upon the final kinetic energies of the neutron and proton within the nucleus, four distinct situations may occur:

i. the neutron escapes and the proton remains within the nucleus;
ii. the proton escapes and the neutron remains within the nucleus;
iii. both neutron and proton remain within the nucleus,
iv. both neutron and proton escape from the nucleus.

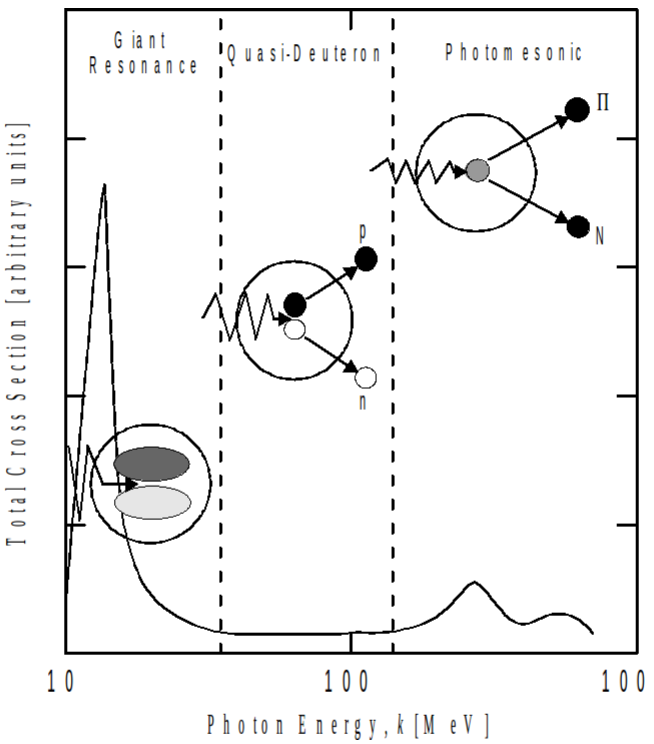

FIG. 1: Showing the primary nuclear photo-interaction according to the incident photon energy. n, neutron; p, proton; N, neutron or proton; $\prod$, mesons π^0, π^- or π^+, $\pi^0\pi^0$, $\pi^-\pi^+$, $\pi^0\pi^-$ or $\pi^0\pi^+$.

Therefore, as a result of the primary interaction, a residual nucleus is formed with probability with a certain excitation energy E^*. In the second stage of reaction, after thermodynamic equilibrium is reached, a process of competition between particle evaporation and fission begins, and in this stage fission may occur with a certain probability $P_f(E^*)$ [4, 5]. Nuclear fissility, or total fission probability, can then be calculated by the product of the mean probability of formation of

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the excited residual nucleus by the probability of fission of this residual, taking into account all possible modes of formation of residual nuclei and the division of incident photon energy by the neutron-proton pair. The fissility of the target nucleus bombarded by a photon of energy k depends, therefore, on the kinematics of the primary interaction, which determines the probability of formation of the residual nucleus at an excitation energy E^*.

The purpose of the present work is to apply the energy and momentum conservation principles and the Pauli Exclusion principle [6] o describe in a simple way the kinematics of photon-nucleus absorption via the process of interaction with quasi-deuterons. As examples, the kinetic energies of nucleons after the primary interaction of photons of 50 and 100 MeV are calculated for two target nuclei, namely ^{27}Al and ^{238}U.

II. KINEMATICS OF THE PRIMARY PHOTOINTERACTION

Let us assume that the photon collides initially with a quasi-deuteron which moves randomly within the nucleus, and the incident photon energy is fully transferred to the two nucleons. Before the collision, in the laboratory reference system (LAB system), the incoming photon, the neutron and proton have energy and momentum represented by k, E_n, E_p and $\mathbf{k}, \mathbf{p}_n, \mathbf{p}_p$, respectively, and in the center of mass reference system (CM system) by k', E_n', E_p' and $\mathbf{k}', \mathbf{p}'_n, \mathbf{p}'_p$. After the collision in the LAB system the neutron and proton move with energy and momentum E_{n*}, E_{p*} and $\mathbf{p}_{n*}, \mathbf{p}_{p*}$, respectively, and E'_{n*}, E'_{p*} and $\mathbf{p}'_{n*}, \mathbf{p}'_{p*}$ in the CM system. Figure 2 shows the primary interaction, where ϕ is the angle between the direction of the incoming photon and the direction of the quasi-deuteron in the LAB system; α_1 and α_2 are the scattering angles of the proton and neutron relative to the direction of the incoming photon in the LAB system, and θ' is the polar angle of the proton in the final state of the interaction in the CM system. The use of the CM system simplifies in a significant way the relativistic treatment of reactions between two or more particles, since the total momentum is null.

The quantity $E^2 - P^2c = M^2c^4$, where E, P and M are, respectively, the relativistic total energy, momentum and rest mass of a system of particles, and is the speed of light in vacuum, is a relativistic invariant [7]. Thus, by using it before the interaction in the LAB system, and after the interaction has occurred in the CM system to the reaction

$$\gamma + (n+p) \rightarrow n^* + p^* \tag{1}$$

we get

$$(k+2T+2mc^2)^2 - (\mathbf{k}+\mathbf{p}_n+\mathbf{p}_p)^2c^2 = (E'_{n*}+E'_{p*})^2 = E'^2 \tag{2}$$

for the square of the total energy available in the CM system, where it was considered here the masses of nucleons approximately equals, $m_s \simeq m_p$ = m, and that their initial kinetic

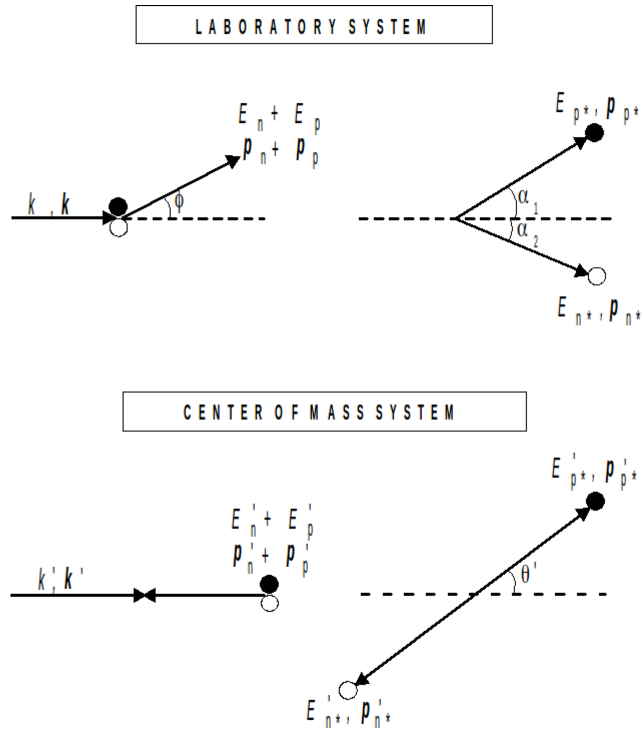

FIG. 2: Showing an interaction of a photon of energy, k, with a quasi-deuteron in the laboratory system (LAB), and in the center-of-mass system (CM). Symbols for the different quantities are defined in the text (section II).

energies in the LAB system are the same,T_n = T_p = T. Given that P and T (kinetic energy) of a particle of mass m are related to each other as

$$P^2c^2 = 2mc^2T + T^2, \tag{3}$$

it results that

$$E'^2 = 2[m^2c^4 + (T+mc^2](T+2k+mc^2) - \mathbf{k}(\mathbf{p}_n+\mathbf{p}_p)c^2 - \mathbf{p}_n\mathbf{p}_pc^2]. \tag{4}$$

In order to obtain the final relativistic energies of the neutron and proton in the LAB system we can use a proper Lorentz transformation [8]. For the proton, for example, the energy-momentum four-vector turns out to be

$$\begin{bmatrix} p_{px^*} \\ p_{py^*} \\ p_{pz^*} \\ iE_p* \end{bmatrix} = \begin{bmatrix} \gamma_c & 0 & 0 & -i\gamma_c\beta_c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\gamma_c\beta_c & 0 & 0 & \gamma_c \end{bmatrix} \begin{bmatrix} p'_{px^*} \\ p'_{py^*} \\ p'_{pz^*} \\ iE'_p* \end{bmatrix} \tag{5}$$

Here β_c is the center of mass velocity and is the gamma factor for the center of mass; the indexes x, y, z denote the directions of a system of orthogonal axes. By considering the incident photon direction as the positive direction of x-axis , from Figure 2 we can write

$$\left.\begin{aligned} p'_{p^*_x} &= p'_{p^*} cos\theta' \\ p'_{p^*_y} &= p'_{p^*} sin\theta' \\ p'_{p^*_z} &= 0 \end{aligned}\right\} \tag{6}$$

So, equation 5 gives

$$\left.\begin{aligned} p_{p^*_x} &= \gamma_c p'_{p^*} cos\theta' + \gamma_c \beta_c E'_{p^*} \\ p_{p^*_y} &= p'_{p^*} sin\theta' \\ p_{p^*_z} &= 0 \\ iE_{p^*} &= i\gamma_c \beta_c p'_{p^*} cos\theta' + i\gamma_c E'_{p^*} \end{aligned}\right\} \tag{7}$$

The velocity of the center of mass of a system of particles is given by the ratio of momentum to total energy:

$$\boldsymbol{\beta_c} = \frac{\mathbf{p}_c}{E} = \frac{(\mathbf{k} + \mathbf{p}_n + \mathbf{p}_p)}{c} k + 2T + 2mc^2. \tag{8}$$

Thus the γ_c factor is written as

$$\gamma_c = \frac{1}{(1-\beta_c^2)^{\frac{1}{2}}} = \frac{1}{[1-(\frac{(\mathbf{k}+\mathbf{p}_n+\mathbf{p}_p)c}{k+2T+2mc^2})^2]^{\frac{1}{2}}}, \tag{9}$$

or, taking into account the expression 5,

$$\gamma_c = \frac{k + 2T + 2mc^2}{E'}. \tag{10}$$

Since in the CM system the magnitude of momenta of the nucleons is the same, i.e.,

$$p'_{p^*} = p'_{n^*} = p'^*, \tag{11}$$

we can write

$$\begin{aligned} E'_{n^*} &= \sqrt{m^2c^4 + p'^2_{n^*}c^2} = \sqrt{m^2c^4 + p'^{*2}c^2}, \\ E'_{p^*} &= \sqrt{m^2c^4 + p'^2_{p^*}c^2} = \sqrt{m^2c^4 + p'^{*2}c^2}, \\ E' = 2E'_{n^*} &= 2E'_{p^*} = \sqrt{m^2c^4 + p'^{*2}c^2}, \end{aligned} \tag{12}$$

or

$$p'^{*2}c^2 = \frac{E'^2}{4} - m^2c^4. \tag{13}$$

Inserting the results (8) to (13) into the last of equations (7), the energy of proton in the LAB system is expressed as:

$$E_{p^*} = a(1 + bcos\theta'), \tag{14}$$

with

$$a = \frac{k}{2} + T + mc^2, \tag{15}$$

and

$$b = \left\{ \left[\frac{1}{c^2} - \left(\frac{2mc^2}{E'} \right)^2 \right] \left[1 - \frac{E'^2}{(k+2T+2mc^2)^2} \right] \right\}^{\frac{1}{2}}. \tag{16}$$

The final energy of the neutron in the LAB system can be obtained analogously by a Lorentz transformation, or more directly by

$$\begin{aligned} E_{n^*} &= k + 2T + 2mc^2 - E_{p^*}, \\ E_{n^*} &= k + 2T + 2mc^2 - a - abcos\theta', \\ E_{n^*} &= a(1 - bcos\theta'). \end{aligned} \tag{17}$$

The kinetic energies of the nucleons in the final state in the LAB system are easily obtained by subtracting from (14) and (17) the nucleon rest mass:

$$T_{p^*} = a(1 + bcos\theta') - mc^2, \tag{18}$$

$$T_{n^*} = a(1 - bcos\theta') - mc^2. \tag{19}$$

III. SHARING OF INCIDENT PHOTON ENERGY

Values for the initial kinetic energies of the nucleons, T, nd the total energy available in the CM system, E', which allow the calculation of final energy of neutron T_{n^*}, and proton T_{p^*}, after the primary interaction with quasi-deuteron. These quantities have been obtained by considering that the target nucleus is a degenerate Fermi gas at zero temperature limit of non-interacting neutrons and protons confined within a spherically symmetric nuclear potential of radius R, and adopting the following simplifying hypotheses:

i. neutrons and protons move randomly in their respective Fermi gases,

ii. the kinectic energy distributions of neutrons and protons before the primary interaction are replaced by their respective average Fermi energies, and the initial kinetic energy of each nucleon is taken constant and given by the average value

$$T_n = T_p = T = \frac{1}{2}\left(\frac{3}{5}E_F^n + \frac{3}{5}E_F^p \right) = \frac{3}{10}(E_F^n + E_F^p), \tag{20}$$

where E_F^n and E_F^p are the respective Fermi energies for neutrons and protons given by

$$E_F^n = \frac{\hbar^2}{2m}\left(\frac{3}{2}\pi^2\rho_n \right)^{\frac{2}{3}}, \tag{21}$$

$$E_F^p = \frac{\hbar^2}{2m}\left(\frac{3}{2}\pi^2\rho_p \right)^{\frac{2}{3}}, \tag{22}$$

where ρ_n is the neutron density and ρ_n is the proton density,

$$\rho_n = \frac{2N}{\frac{4}{3}\pi R^3}, \tag{23}$$

$$\rho_p = \frac{2Z}{\frac{4}{3}\pi R^3}, \tag{24}$$

m is the nucleon rest mass and $\hbar$ is the Planck's constant divided by 2π [9]. In the above equations the nuclear radius is calculated as R = $(5/3)^{1/2}$ RMS, where values of RMS (root-mean square radius) are those listed in [10].

Hypothesis i above allow us to use for E'^2 the mean value of the expression (4)

$$\overline{E'^2} = 2[m^2c^4 + (T+mc^2)(T+k+2mc^2)], \tag{25}$$

which simplifies the calculations, since it makes dispensable the knowledge of more details about the primary interaction, such as the angle ϕ between the direction of the incident photon and the direction of the quasi-deuteron.

From the conservation of energy applied to the reaction γ + (n + p) $\rightarrow n^*$ + p^* in the LAB system we can write

$$k + 2T + 2mc^2 = E_{n^*} + E_{p^*},$$

or

$$T_{n^*} + T_{p^*} = k + 2T = k + \frac{3}{5}(E_F^n + E_F^p), \tag{26}$$

which is valid for any value of angle θ'.

According to the Pauli Exclusion principle [6] the final energies of the nucleons can not be less than their respective Fermi energies:

$$\left.\begin{array}{l} T_{n^*} > E_F^n \\ T_{p^*} > E_F^p \end{array}\right\} \tag{27}$$

Thus, the minimum value of kinetic energy of neutron after the interaction is E_F^n and the maximum value is obviously the one for which the kinetic energy of proton is minimum T_{p^*} = E_F^p:

$$T_{n^*_{min}} \leq T_{n^*} \leq T_{n^*_{max}}, \tag{28}$$

where

$$T_{n^*_{min}} = E_F^n, \tag{29}$$

$$T_{n^*_{max}} = k + (3E_F^n - 2E_F^p)/5. \tag{30}$$

The extreme values of the proton polar angle θ' n the final state in the CM system are obtained from equation (19) with T_{n^*} = E_F^n, and from equation (18) with T_{p^*} = E_F^p:

$$\theta'_{min} = \arccos\left[\frac{1}{b} - \frac{E_F^n + mc^2}{ab}\right] \tag{31}$$

$$\theta'_{max} = \arccos\left[-\frac{1}{b} + \frac{E_F^p + mc^2}{ab}\right]. \tag{32}$$

According to the Pauli principle, the primary interaction can only take place if $T_{n^*} > E_F^n$ and $T_{p^*} > E_F^p$. Therefore, from the expression (26) it can be easily verified that the model described above can only be applied to a given nucleus when the incoming photon energy is greater than a certain threshold energy value given by

$$k \geq \frac{2}{5}(E_F^n + E_F^p). \tag{33}$$

In the case of ^{27}Al target nucleus E_F^n = 28.9 MeV and E_F^p = 27.5 MeV [11], so that the incident photon energy must be at least 22.6 MeV to reaction (1) occur; for ^{238}U target nucleus, where E_F^n = 37.3 MeV and E_F^p = 27.4 MeV [4], the incident photon energy must be at least 25.9 MeV. In Figs. 3 and 4 are shown examples of the division of the incoming photon energy, k, between the neutron-proton pairs of the target nuclei ^{27}Al and ^{238}U as a function of the proton polar angle in the CM, θ', for k = 50 MeV and k = 100 MeV.

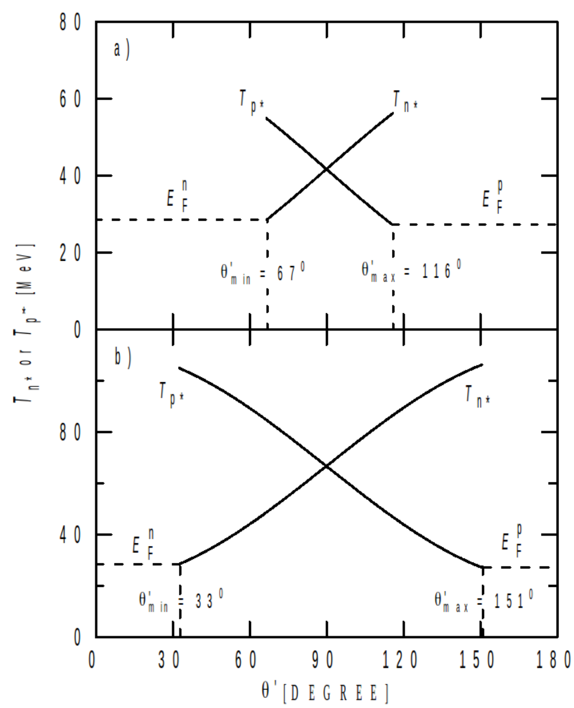

FIG. 3: Division of incident photon energy, k, between the neutron-proton pair of a target nucleus of ^{27}Al as a function of the proton polar angle θ' in the CM system. The limits of θ' allowed by the Pauli exclusion principle are indicated; a) k = 50 MeV; b) k = 100 MeV.

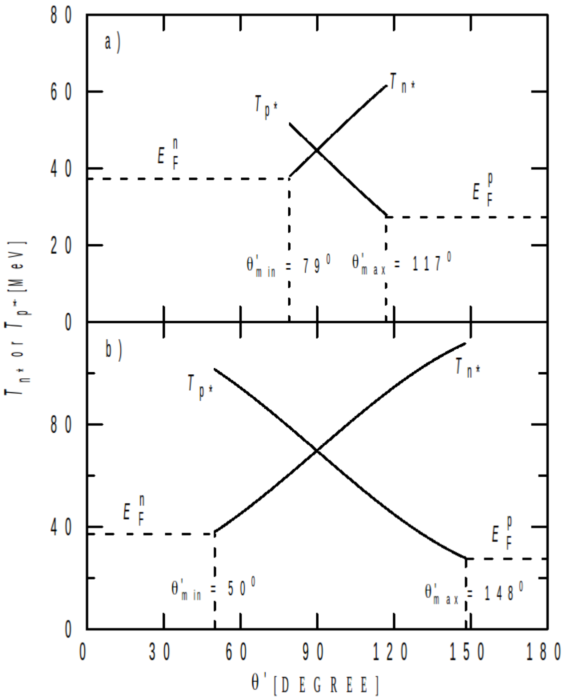

FIG. 4: The same as in Fig. 3, but for a target nucleus of ^{238}U .

IV. FINAL REMARKS AND CONCLUSION

The interaction of photons with nuclei in the energy range from the end of giant resonance up to the threshold for pion production can be understood on the basis of a model in which the primary photo-interaction is considered to occur with neutron-proton pairs, as described by Levinger [2, 3]. The relativistic energy-momentum conservation laws and the Pauli Exclusion principle were applied to obtain the energies of the nucleons after the primary interaction. It was supposed that the target nucleus is described by non-interacting Fermi gases of neutrons and protons confined within a spherically symmetric nuclear potential. An approximation used in the present study was to consider equal masses for protons and neutrons, and that, initially, the proton and neutron move randomly within the nucleus with constant kinetic energies given by their respective average Fermi energies.

As examples of application of the model described in the above lines, the sharing of incident photon energy between nucleons of a n-p pair has been obtained for a light nucleus (^{27}Al) and a heavy nucleus (^{238}U) in two different energies of the incident photon, namely, 50 and 100 MeV.

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