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Approximation Algorithms for the Bin Packing Problem

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Abstract

The Bin Packing Problem is a classical NP-hard problem. This study gathers the proofs of the approximation ratios for several known approximation algorithms for this problem, including Next-Fit, First-Fit, and two approximation schemes. It also considers the inapproximability of the Bin Packing Problem by ratios lower than 1.5, and lower bounds for the competitive ratios of online algorithms, bounded-space and otherwise.

Key words:

packing, approximation, optimization

Introduction

The Bin Packing Problem models the logistic problem of finding the minimum amount of bins needed to pack a certain set of items. It has several applications, such as optimizing the usage of storage space for products or digital files, scheduling tasks to several machines, or cutting pieces of fabric, glass, metal and other materials.

The Bin Packing Problem is NP-hard and thus can't be solved optimally in polynomial time unless P = NP, a long-standing conjecture in Computer Science. This study focuses on approximation algorithms for the problem, that is, polynomial-time algorithms with a guarantee that the value of their solutions will be always less or equal than $\alpha OPT(L)$ for some $\alpha > 1$, where $OPT(L)$ is the value of an optimal solution for instance L. The constant α is called the approximation ratio of the algorithm.

The algorithms studied also include online algorithms, which are algorithms that must decide in which bin to put an item without any knowledge about the next items on the instance.

Results and Discussion

The general results about approximations for the Bin Packing Problem studied in this project include the proof of inapproximability of the problem by a ratio smaller than 1.5.[1] Despite that, algorithms for this problem can still have lower asymptotic approximation ratios ρ, that is, the value of their solutions is always less than equal than $\rho OPT(L) + c$ for some constant $c > 0$.

Online algorithms also cannot have a ratio lower than 1.5,[2] and bounded space online algorithms, that is, algorithms that always keep no more than a constant amount of bins available to pack items, cannot have a ratio lower than 1.6910.[3]

The classical online algorithms studied were: Next Fit, where an item is put either in the last opened bin, if it fits, or opens a new bin for the item; and First Fit, where an item is put in the first opened bin where it fits, and a new bin is only opened when the item doesn't fit any of the current ones. The value of Next Fit solutions is $NF(L) \leq 2OPT(L) + 1$,[4] and the value of First Fit solutions is $FF(L) \leq (17/10)OPT(L) + 2$.[5]

Another algorithm studied was First Fit Decreasing, with a solution of value $FFD(L) \leq (11/9)OPT(L) + 4$.[6]

After that, we studied two Asymptotic Polynomial Time Approximation Schemes (APTAS), classes of algorithms with an asymptotic approximation ratio of $1+\varepsilon$ for some $\varepsilon > 0$. The APTAS proposed by La Vega and Lueker in 1981[7] returns solutions of value less or equal than $(1+\varepsilon)OPT(L) + 1$, and the APTAS proposed by Karmarkar and Karp in 1982[8] returns solutions of value less or equal than $(1+\varepsilon)OPT(L) + O(\log^2 SIZE(L))$, where $SIZE(L)$ is the sum of the sizes of every item in L.

Conclusions

All the results mentioned and their proofs were gathered in a technical report. A continuation of this research is planned for variants of the Bin Packing Problem, such as n-Dimensional Bin Packing, where the size of items is an n-tuple instead of a single value, and Min-Sum Bin Packing, where the cost of packing items in the first bins of the solutions is lower than the cost of packing them in the last ones.

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